

**INVENTORY MODEL WITH VARIABLE HOLDING COST TO  
ACCOMMODATE PLANNED SHORTAGE  
(Study Case at CV. Yopan Ceramics)**

**THESIS**

**Submitted to International Program**

**Department of Industrial Engineering in Partial Fulfillment of  
The Requirement for the degree of Sarjana Teknik Industri at**

**Universitas Islam Indonesia**



**By**

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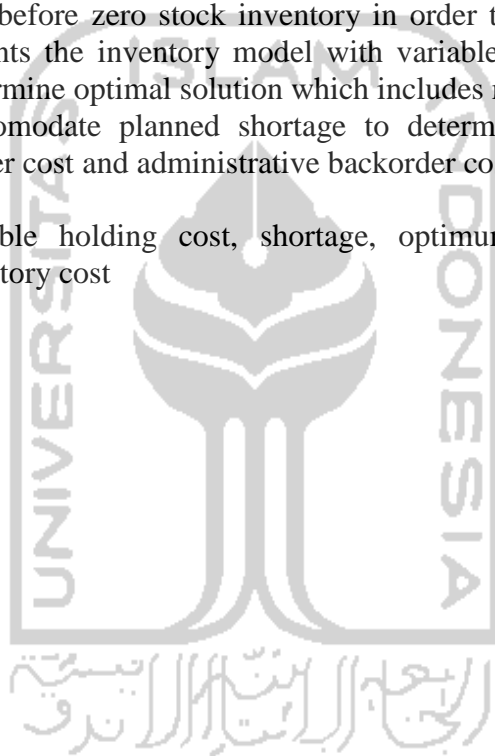
**INTERNATIONAL PROGRAM  
DEPARTMENT OF INDUSTRIAL ENGINEERING  
UNIVERSITAS ISLAM INDONESIA  
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## ABSTRACT

Inventory has become the biggest problem and needs to get extra attention. The inventory problem is associated with products which are stored in the storage for longer term and the higher cost needed to maintain it. In the real problem, the storage cost/holding cost is variable dependent to storage time used and to be increasing function of the time spent in the storage. Two types of variable holding cost are retroactive holding cost and incremental holding cost. Those type created with the purpose of determining optimal quantity ,cycle time, and minimum inventory cost which includes of demand, order cost, demand elasticity to inventory, and holding cost. Based on common real-life condition that sometimes the demand is greater than the inventory level. This condition caused the inventory shortage and anticipated by doing backorder before zero stock inventory in order to meet customer satisfaction. This paper presents the inventory model with variable holding cost using a simple algorithm to determine optimal solution which includes minimizing total inventory cost and also to accomodate planned shortage to determine the shortage cost which includes backorder cost and administrative backorder cost.

Keywords: variable holding cost, shortage, optimum solution, backorder, total inventory cost



**This thesis, I dedicate to my parents,**

**Thanks to my Dad and Mom for your prayer**

**sacrifice and advices.**



## MOTTO

*“O you, who have believed, seek help through patience and prayer. Indeed, Allah is with the patient.” (Al-Baqarah: 153)*

*“There is no problem that can not be solved as long as there commitment to resolve”*

*"Allah has educated me with the best of education."*

*(HR. by Ahmad)*





## PREFACE



### **Assalamualikum Wr.Wb**

Alhamdulillah robbil a'lamin. First of all, I would like to say thanks to Allah SWT that always give us bless and mercy for all mankind who had faith and worship to God. Because of His bless and mercy, I can finish my thesis with Title **INVENTORY MODEL BASED WITH VARIABLE HOLDING COST TO ACCOMODATE PLANNED SHORTAGE**. This Thesis is part of requirement to get bachelor's degree of Industrial Engineering Department at University Islam Indonesia.

I would also like to express my profound gratitude for the love, support, and faith to my parents, Nurjono, SH and Puji Rahayu. And I'm so thankful for the happiness brought by my sisters, Galuh Hayu Noormalisa, and my brother, Rizda Noor Ali Wicaksono and Swastika Noor Yudha Pratama

In this opportunity I would like to say special thanks and deepest respect to my supervisor Dr. Mirwan Ushada, S.TP, M.App.Life.Sc that always gives support, advice, guidance, and suggestion to me so that I can finish my thesis. Furthermore, my appreciation goes to CV. Yopan Ceramic for the cooperation as my research object.

At last, I would like to thank all my friends in FIT UII, especially IP FIT student's year 2007 for the great experience and opportunity during our school year.

**Wassalamualaikum Wr. Wb**

Yogyakarta, February 30, 2012



Author

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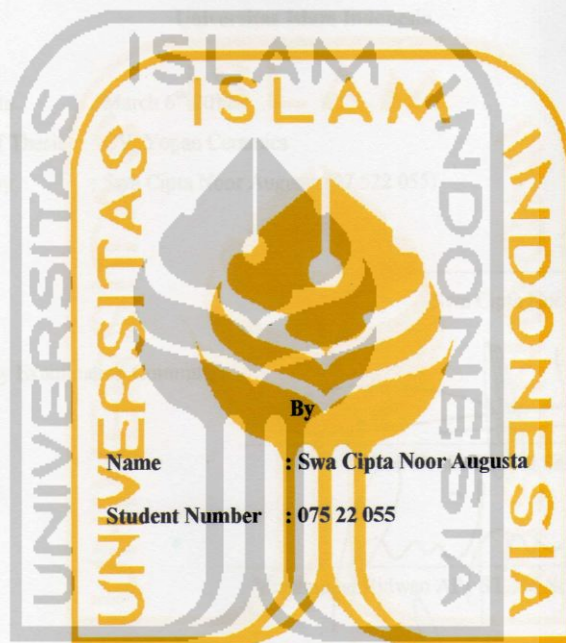
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THESES  
**THESIS APPROVAL OF SUPERVISOR**  
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ACCOMMODATE PLANNED SHORTAGE**

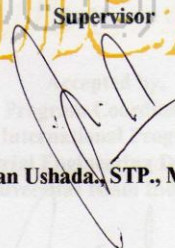
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## APPENDIX

### Appendix 1. Inventory Condition in CV. Yopan Ceramics

Periode	demand	Production	Inventory	Condition
Oct-10	213	266	53	
Nov-10	209	275	66	
Dec-10	297	282	-15	shortage
Jan-11	213	269	56	
Feb-11	314	295	-19	shortage
Mar-11	196	253	57	
Apr-11	294	283	-11	shortage
Mei-11	287	269	-18	shortage
Jun-11	217	283	66	
Jul-11	212	287	75	
August-11	281	263	-18	shortage
Sep-11	292	275	-17	shortage

### Appendix 2. Secant method

Secant method (  $Q=1670, Q=2369$ )

x(0)	x(i)	f(x(0))	f(x(i))	term	apx. error	round
1670	2369	-545427,26	63711,33275	2295,889837	29,50612073	29,50612
2369	2295,889837	63711,33275	8450,587216	2284,70968	3,184393321	3,18439
2295,889837	2284,70968	8450,587216	-129,4555102	2284,878366	0,48934694	0,48935
2284,70968	2284,878366	-129,4555102	0,264100873	2284,878023	0,007382712	0,00738
2284,878366	2284,878023	0,264100873	8,2578E-06	2284,878023	1,50307E-05	0,00002

Secant method (  $Q=2369, Q=2978$ )

x(0)	x(i)	f(x(0))	f(x(i))	term	apx. Error	round
2369	2978	63779,08683	647554,2756	2302,465	20,44997	20,44997

### Appendix3.Scientific analysis for retroactive and incremental

#### a. changed value $\beta$

changed value	%	SENSITIVITY ANALYSIS (RETROACTIVE)			
		Q*	T*	TIC*	%
0,1	0,00%	2068	0,35	1653670,913	0,00%
0,12	20,00%	2203	0,32	1762312,314	6,57%
0,14	40,00%	2351	0,3	1880026,391	13,69%
0,16	60,00%	2510	0,28	2007707,472	21,41%
0,18	80,00%	2683	0,26	2146350,374	29,79%
0,2	100,00%	2872	0,24	2297061,128	38,91%

changed value	%	SENSITIVITY ANALYSIS (INCREMENTAL)			
		Q*	T*	TIC*	%
0,1	0,00%	2369	0,4	1218980	0,00%
0,12	20,00%	2755	0,4	1286054	5,50%
0,14	40,00%	3225	0,4	1366693	12,12%
0,16	60,00%	3801	0,4	1464388	20,13%
0,18	80,00%	4513	0,4	1583658	29,92%
0,2	100,00%	5400	0,4	1730434	41,96%

#### b. changed value H

changed value	%	SENSITIVITY ANALYSIS (RETROACTIVE)			
		Q*	T*	TIC*	%
3,6,9	-25,00%	2406	0,4	1443004,137	-12,74%
4,8,12	0%	2068	0,35	1653670,913	0,00%
5,10,150	25,00%	1839	0,31	1838035,294	11,15%
6,12,18	50,00%	1670	0,29	2003829,196	21,17%
7,14,21	75,00%	1540	0,27	2155620,689	30,35%
8,16,24	100,00%	1436	0,25	2296372,098	38,87%

changed value	%	SENSITIVITY ANALYSIS (INCREMENTAL)			
		Q*	T*	TIC*	%
3,6,9	-25,00%	2369	0,4	1106764	-9,21%
4,8,12	0%	2369	0,4	1218980	0,00%
5,10,150	25,00%	2369	0,4	1331196	9,21%



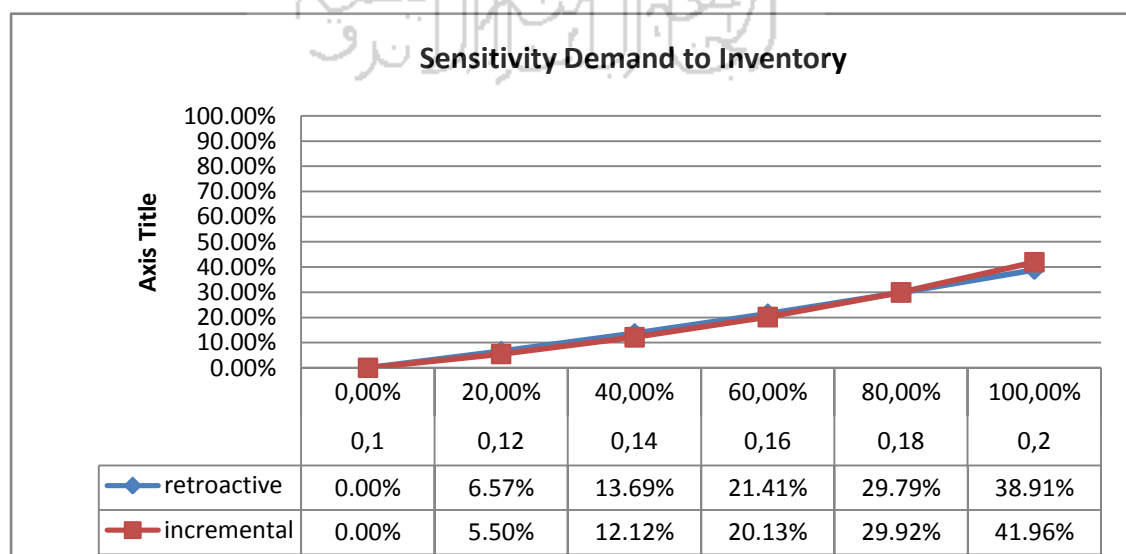
changed value	%	SENSITIVITY ANALYSIS (INCREMENTAL)			
		Q*	T*	TIC*	%
6,12,18	50,00%	2369	0,4	1443412	18,41%
7,14,21	75,00%	2369	0,4	1555627	27,62%
8,16,24	100,00%	2369	0,4	1667843	36,82%

**c. changed value T**

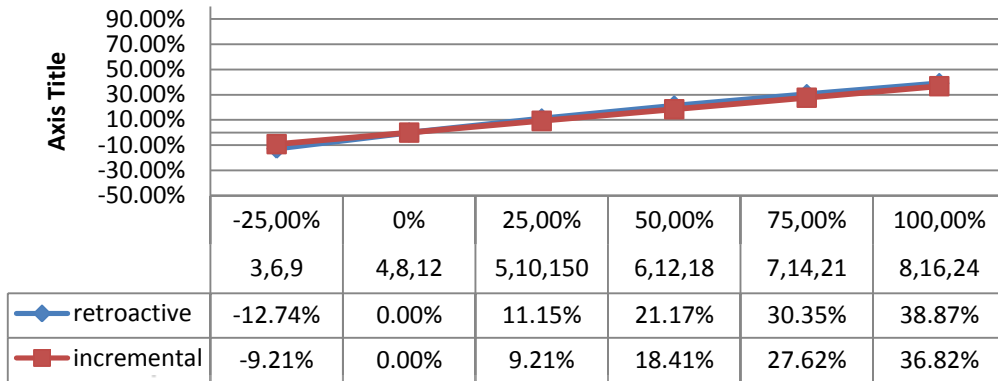
changed value	%	SENSITIVITY ANALYSIS (RETROACTIVE)			
		Q*	T*	TIC*	%
0,2 0,4 ∞	0,00%	2068	0,35	1653670,913	0,00%
0,25 0,5 ∞	25,00%	1405	0,2	1498208,272	9,40%
0,3 0,6 ∞	50,00%	1839	0,31	1838035,294	11,15%

changed value	%	SENSITIVITY ANALYSIS (INCREMENTAL)			
		Q*	T*	TIC*	%
0,2 0,4 ∞	0,00%	2369	0,4	1218980	0,00%
0,25 0,5 ∞	25,00%	3035	0,5	1335142	9,53%
0,3 0,6 ∞	50,00%	2285	0,3	1394019	14,36%

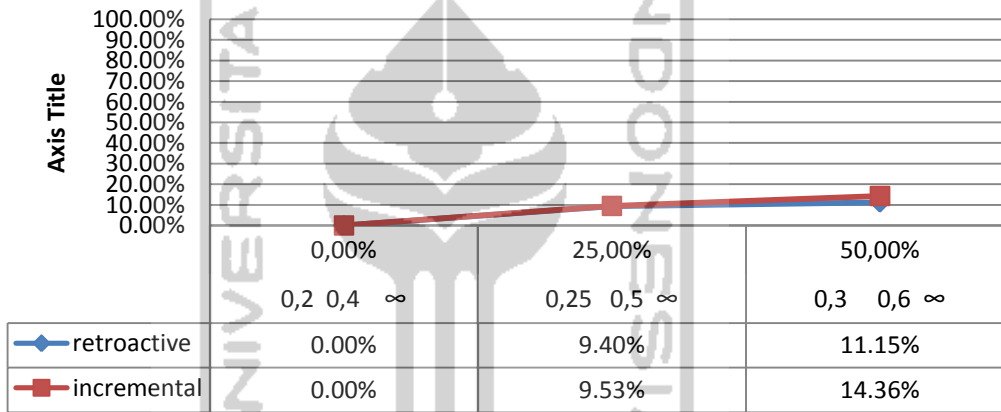
**Appendix4.Change Value Graph**



### Sensitivity Variable Holding Cost



### Sensitivity Period Time



## CHAPTER I

### INTRODUCTION

#### 1.1 Background

The real world observation shows that the product availability is greater than sales, and this creates inventory. In a company, inventory has become the biggest problem and needs extra attention. The inventory problem is associated with products which are stored in the storage for longer terms. Therefore, a company must have investment costs to store the product. The longer the time to store the material in the storage, the higher the cost needed to maintain it (Alfares, 2007).

Most of companies have limitations in space to store the product in the storage. Thus, the inventory level must be organized well in order to reduce the cost of storing the material. Many researchers have been trying to conduct research related to controlling inventories. Chung (2003) has studied about the inventory level which can be developed by an algorithm to find the optimal order quantity. The study aims to reduce the inventory cost based on optimal order quantity.

In the real problem, the holding cost increases since longer time is needed in the storage. It means that holding cost is variable dependent to storage time used (Singh, et.al., 2010). There are two types of time-dependent holding cost which are retroactive and incremental. Retroactive means the unit of the holding cost rate of the last storage applied to all storage time otherwise incremental is the holding cost rate of each period, including the last period applied only to unit stored in particular period. Alfares (2007) considers the variable holding cost which is higher for longer storage

time in the inventory model to determine optimal inventory policy related to cycle time .

Many researchers assume the inventory model without any shortages occurred. It means that the company can fulfill the demand. In fact, this condition is not represent the real system (Shamsi, et.al., 2010). The demand uncertainty will cause the difficulties in controlling the inventory. Sometimes the demand is greater than the inventory level. Therefore, shortages is occurred (Hu and Liu, 2010). To overcome this problem, the shortage of inventory must be anticipated in order to meet customer satisfaction by doing backorder before zero stock inventory.

This research concerns with the case on a CV. Yopan Ceramics, that has less space to store the material. This company has high order and production quantity so it requires much space to store the material and it will impact to the higher inventory cost. In addition, the problem is not only less space of warehouse but also the number of demand exceeds the capacity. Therefore, CV. Yopan Ceramics needs more service in organizing the inventory associated with determining the optimal quantity (order and production) and cycle time. Meanwhile, the optimal quantity is being ordered or produced, it does not always meet the customer demand because of its uncertainty and this causes shortage condition occurred. Thus, the company needs to accommodate planned shortage in order to respond the customer requirement.

To overcome this problem, this research proposes a development of inventory model by combining Alfares (2007)'s model to determine the optimal quantity and cycle time as decision variable to minimize inventory cost and Shamsi, et.al., (2010) by considering the shortage that consist of time dependent and time independent shortage cost.



## 1.2 Problem Formulation

Based on the above background, the problem formulated could be seen as follow:

- a. How is the model of variable holding cost in retroactive and incremental?
- b. How to accomodate planned shortage?
- c. How is the influence of variable holding cost to inventory cost?

## 1.3 Research Limitations

A limitation of the research is required to focus the research being conducted so that the goal can be achieved. The boundaries of the research are:

- a. The research object is focused on CV. Yopan Ceramic as ceramic producer
- b. This research concern on optimal order quantity for planned shortage.
- c. The level of accuracy of the output resulted from the implementation of the proposed model firmly depends on the input or data provided by the company.

## 1.4 Research Objective

The objectives of this research are:

- a. To compare the result of retroactive and incremental as effect of inventory cost
- b. To accomodate planned shortage
- c. To determine the influence of variable holding cost to inventory cost

## 1.5 Research Significance

The significance of this research is:

- a. Implementation of inventory model based on real world problem
- b. To be evaluation for company in controlling the inventory

## **1.6 Thesis Structure**

The thesis structure is as follows:

### **CHAPTER II LITERATURE REVIEW**

Literature review provides information on previous studies. The objective is to seek the novelty of this research. Besides, it also explains the background theory.

### **CHAPTER III RESEARCH METHODOLOGY**

This chapter will present the research methodology, model development and the data necessary. Furthermore, this chapter will explain about the techniques of data collection and analysis. The final section of this chapter contains the framework of the research.

### **CHAPTER IV DATA COLLECTION AND PROCESSING**

This chapter presents information of data that have been collected during the research. It also contains problem solving using the proposed model or tools that are implemented in the data processing as well the analysis using the proposed model.

### **CHAPTER V DISCUSSION**

This chapter provides a discussion after data analysis. Furthermore, it also discuss about the result in order to see the ability of the proposed model to overcome the problem.

## CHAPTER VI CONCLUSION AND RECOMMENDATIONS

The conclusion and recommendations for further research will be described in this chapter.

### REFERENCES

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Table

Figure



## CHAPTER II

### LITERATURE REVIEW

#### 2.1 Previous Research

Research issue concerning inventory problem has been studied by many researchers. Chang and Dye (2000) constructed an inventory model considering stock dependent demand to determine the optimal ordering period to maximize total cost savings. In real world, the demand rate is usually influenced by the amount of the stock level. Thus, the demand rate may grow up or down with the on-hand stock level and inventory level dependent demand can be developed with an algorithm to find optimal order quantity for reducing inventory cost ( Chung, 2003)

Most of previous models assume the holding cost to be constant for the entire inventory cycle but in real condition, the storage time can be varied with time. Thus, the holding cost can be variable based on the storage time that used (Singh, et.al., (2010). Shao, et.al., (2000) considered a variable holding cost to determine the optimal target in order to maximize expected profits. In real situation, manufactures may hold goods that have been rejected by one customer and then to sell that goods to another customers in the same market at later date. Thus, the cost for holding the goods will increase until the good sold.

Alfares (2007) explored inventory model which the demand rate depends on the inventory level and the product availability are greater than sales. Thus, the holding cost is variable and become an increase function for longer storage periods. This paper presents representation of real situation in which the storage times can

beclassified into a different range. There are two types of time-dependent holding cost which are retroactive and incremental. Retroactive means the unit of the holding cost rate of the last storage applied to all storage time otherwise incremental is the holding cost rate of each period, include the last period applied only to unit stored in particular period and solved using numerical method. Those types are developed for determining the optimal order quantity and the optimal cycle time to minimize total inventory cost (TIC).

The demand uncertainty will cause the difficulties in controlling the inventory. Sometimes the demand is greater than the inventory level. Therefore, shortages are occurred (Hu and Liu, 2010). Xu and Sarker (2003) developed model in the effect of shortage cost in inventory system. The shortage has significant effect on the design of inventory system and are also given importance in some special cases to decide which option to be followed between reducing the production rate, cycle time, or combines options 1 and 2 together to avoid breaking the shelf-life constraint.

Chen and Lo (2006) considered allowable shortage in the optimal production length and time length when backorder is replenished for the imperfect production. The system of imperfect production with the product sold with free minimal repair warranty is formulated as a cost minimization model to find the optimal solutions.

Cheng, et al., (2007) extended Chen and Lo (2006) approach that allows shortage for product life cycle in replenishment policies to determine the number of inventory replenishment, the inventory replenishment time points, and the beginning time points of shortage by minimizing the total cost of the inventory replenishment. Shamsi, et al., (2010) presented a model by considering shortage cost for imperfect quality items and the result shows indicate the model is very sensitive to

shortage cost. Therefore, if the existence of shortages are ignored, then the obtained results may differ considerably from the optimal outcome.

To overcome this problem, this research proposes a development of inventory model by combining Alfares (2007)'s model to determine the optimal quantity and cycle time as decision variable to minimize inventory cost and Shamsi, et.al., (2010) by considering the shortage that consist of time dependent and time independent shortage cost.

## **2.2 Basic Theory**

### **2.2.1 Inventory philosophy**

Inventory is a stock of materials that used to facilitate in production process and to satisfy customer demand (Schroeder, 1994). The existence of inventory in company need to be regulated such as the process of fullfilling the customers need can be guaranteed and the incidence of the idle resources and time in waiting further processing can be removed, this is caused to be more efficient. According to Sumayang (2003), inventory is a material saving that consist of raw materials, work in process and finished product. Based on this definition, inventory can be summarized as materials (raw materials, work in process and finished roduct) that must be stored and maintained in warehouse in order to always ready to fulfill the company need.

Basically, inventories simplify the operation of a company that should be done to produce the goods and further to consumers. In general, inventories exist because there is an imbalance between the supply of an item at a location and its consumption or sale there. The imbalances are the consequence of many technical, economic, social,

and natural forces. Note that inventories are a consequence and not a cause of some policy or action. Hence inventories become a dependent rather than an independent variable. According to Assauri (1993), as for reasons inventory is needed by a company because:

1. There is time needed to complete the production operation to move products from one process to another process.
2. To allow one unit to the operating schedule freely, independently of the others.

Inventory is one of the most active elements in the company's operations. Therefore, the availability of sufficient inventory will ensure the sustainability of the company's operations because of the time between one process to the next process can be minimized.

### **2.2.2 Type of Inventory cost**

Inventory cost is one of the key factors in determining an inventory policy. The goal of inventory policy is to get the amount of raw materials at the right place, right time, with low cost (Tersine, 1994). Most models for planning inventory requirements consider costs of various types. According to Tampubolon (2004) the cost caused by the existence of inventory are ordering cost, holding cost, and shortage cost.

### **2.2.3 Ordering Cost**

Ordering costs are costs associated with ordering product (material) from the supplier. Because the order is made then the materials that were ordered are delivered to the inspection at the warehouse. The cost for this is constant and the cost does not depend on the size or amount of goods ordered (Tersine, 1994). In this case, the ordering

costs included in all costs related to an order of goods, including administrative costs, placement of order purchasing, revenue cost, inspection cost, and doing the processing necessary to complete the transaction

#### **2.2.4 Holding Cost**

Holding cost is those required cost because of the inventory that includes all costs incurred by the company as a result of the inventory. Those costs are related to the average level in inventory that is always there in the warehouse, so the amount of this cost varies, depending on the average amount of inventory contained in the warehouse. The costs included in the holding cost are all costs incurred because the goods are stored including capital costs, insurance cost, storage cost, shrinkage, obsolescence, deterioration, and transportation.

Capital cost reflects the stock of goods in storage. Therefore, costs for having inventory must be calculated in the cost of inventory systems. Insurance coverage requirements are dependent on the amount to be replaced if property is destroyed. The material (goods) need a place to store that product, thus a company will spend the cost for storage cost. If the storage is rented so it will be called rent cost, but if the company has its own storage, it is called depreciation cost. Shrinkage is the decrease in inventory quantities over time from loss or theft and usually measured from the based on percentage of loss quantities. Obsolescence is the risk that an item will lose value because of shifts in styles or consumer preferences and obsolescence costs are usually measured by the decreasing amount of the sale value of the goods. Deterioration means a change in properties due to age or environmental degradation.



Many of the models developed based on that assumption that will be used for calculating inventory holding costs and the goal is to minimize the average annual operating costs. In line with that assumption is the practice of establishing the holding cost of inventory items as a percentage of their dollar value (Tersine, 1994).

### 2.2.5 Shortage Cost

Shortage costs are those costs which are caused because inventories are smaller than the amount required, such as losses or additional costs are required because of a customer demand or order of goods while the materials (goods) required are not available.

In this situation, the companies will have two possible options that are the demand will be canceled completely or goods that are still less will be filled later. Basically, the company will not choose the first operation that the demand will be canceled completely, because it will eliminate the customer's sympathy and will affect the company's image. Second option, the goods are still less and will be fulfilled at next production cycle. Thus, there are two types of stock out (shortage) that are:

1. **Lost Sales Cost**, the costs caused by inventory shortages, thus customers to cancel the order. The cost is balanced with the benefits or profits to be obtained from the sale of these products.
2. **Back Order Cost**, occurs when consumers are still willing to wait until the order is filled, so in this case the sale is not lost but just delayed. These costs represent costs spent to process the orders, and additional transportation costs if the order does not seem to be distributed through normal distribution.

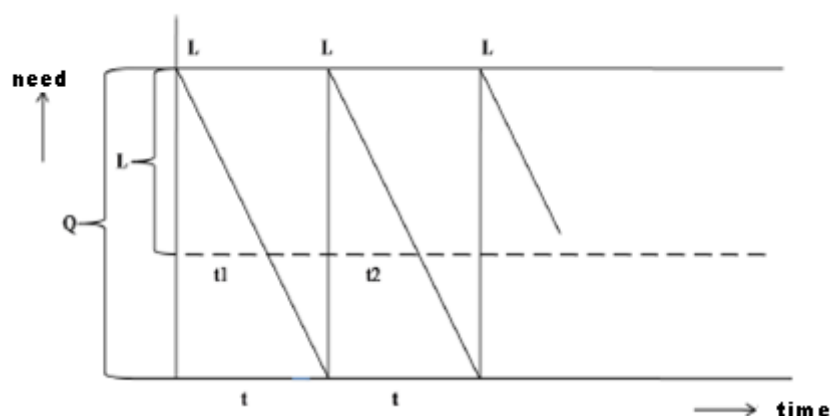


Figure 2.1 Condition of Lost Cost (Rangkuti, 2000)

According Schroeder (1994) inventory shortage costs can be measured from:

1. The quantity that cannot be fulfilled

Usually measured by lost profits because the manufacturer cannot meet the demand or from losses due to interruption of the production process. This condition is termed as a penalty cost.

2. Fulfillment time

The duration of an empty warehouse means the duration of the production process is stopped or duration of a company may not be profitable, so that the idle time can be defined as the missing money. The cost of fulfillment time is measured based on the time required to fulfill a warehouse.

3. Emergency cost

In order for consumers not to be disappointed, it can be done which usually causes an emergency procurement cost greater than the normal procurement. Excess cost compared

to the normal procurement can be used as a measure to determine the unit cost inventory shortage for example Rp/each time the shortage

### 2.2.6 Inventory Model

Inventory model will depend on the type of material, whether these materials are independent or dependent. The Independent demand is influenced by market conditions where outside the control of operating functions, and therefore it is independent to the operating functions otherwise the dependent demand related to the demand for other goods and are independently determined by the market (Schroeder, 1994)

According to Tersine (1994) inventory demand can be divided into two kinds that are: independent demand inventory, the inventory amount is not influenced by an amount of other inventory and dependent demand inventory, the inventory amount is influenced by the amount of other goods inventory.



Figure 2.2 Independent Inventory and Dependent inventory (Sumayang, 2003)

Independent demand and dependent demand show patterns of demand that are very different.

- a. In the independent demand inventory system, the appropriate model is the addition of inventory to the amount used or replacement. At the time, inventory is reduced, this condition stimulates to immediately make reservations in exchange for inventory that has been used.
- b. In a dependent demand inventory system, when the inventory is reduced, the order cannot be done. Ordering will be done when there is demand for goods from the further process steps.

### 2.2.7 Mathematical Model

Mathematical modeling is a process that comprises three phases as follows (Pamuntjak, et.al., 1990):

- a. Formulation of mathematical models
- b. Completion and/or analysis of mathematical models
- c. Interpreting the results to real situations

Here is the process of formulation of the phenomenon/behavior of the real condition in mathematical form. Step in modelling, real condition problems are illustrated in the following diagram:

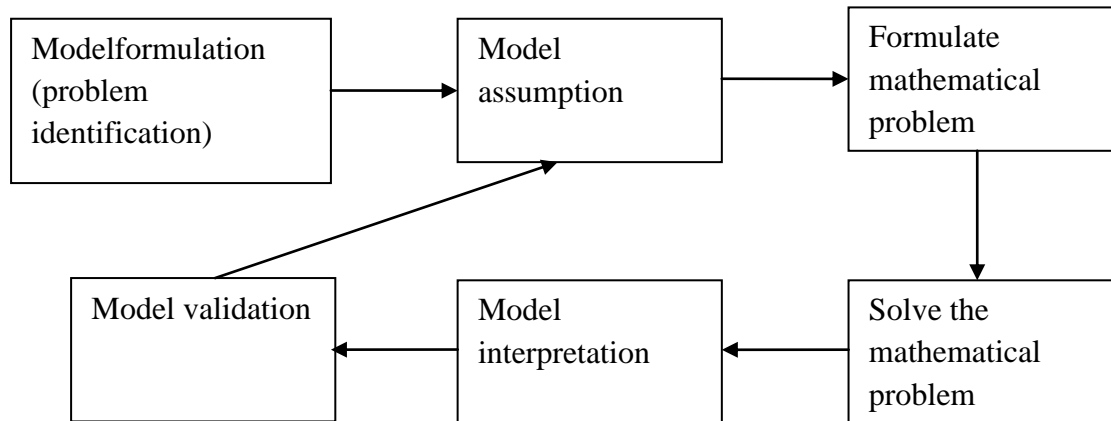


Figure 2.3 Steps in modelling (Baiduri, 2002).

The next modeling steps can be classified as follows:

1. Identify the Problem
2. Making Assumptions
3. Complete or Interpretative Model
4. Verification Model

In this study, the mathematical model used is a mathematical model formulated by Alfares (2007) and Shamsi, et.al. (2010), as described below:

### 1 Retroactive holding cost increase

In a retroactive holding cost, there are 3 solution algorithms to determine the optimal order quantity ( $Q^*$ ), cycle time ( $T^*$ ), and Total Inventory Cost ( $TIC$ )

#### 1a. Optimal order quantity ( $Q^*$ )

$$Q^* = \left[ \frac{KD(1-\beta)(2-\beta)}{h_i} \right]^{1/(1-\beta)} \quad (2.1)$$

1b. Cycle time ( $T$ )

$$T = \frac{Q^{1-\beta}}{D(1-\beta)} \quad (2.2)$$

1c. Total Inventory Cost ( $TIC$ )

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{D(2-\beta)T} \quad (2.3)$$

## 2. Incremental holding cost increase

In Incremental holding cost, an optimal solution algorithm for the optimal order quantity ( $Q^*$ ), quantity on hand ( $Q$ ), and cycle time ( $T$ ) based on the optimum solution of Total Inventory cost ( $TIC$ )

2a. Optimal order quantity ( $Q^*$ )

$$Q^* = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i Q}{(2-\beta)} + \sum_{i=1}^{e-1} (h_{i+1} - h_i) [Q^{1-\beta} - D(1-\beta)t_i]^{1/(1-\beta)} \quad (2.4)$$

$$- \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(1-\beta)t_i]^{(2-\beta)/(1-\beta)} = 0$$

2b. Total Inventory Cost ( $TIC$ )

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} \quad (2.6)$$

## 3. Planned Shortage Cost

In planned shortage models there can be both time-dependent and time-independent shortage costs

3a Time dependent shortage cost

$$TC_{\bar{B}} = C_s \times \bar{B} \quad (2.7)$$

3b. Time Independent shortage cost

$$TC_B = N \times C_b \times B \quad (2.8)$$

### 2.2.8 Numerical method

Numerical method or iterative method are scientific in the sense that represent systematic techniques for solving mathematical problems because the problem cannot be solved efficiently using analytical technique (Chapra and Canale, 1990). There are many numerical methods for solving the minimization problem. Some of the numerical methods for finding roots of equation or solving differential equations require the user to specify initial guesses or starting point. These numerical methods process from an initial guess  $x_0$ , determine a direction in which to search for a better approximation to a solution, construct a sequence of points  $x_1, \dots, x_i$  and terminate when  $x_i$  is sufficiently close to a solution.

The general Quasi-Newton iteration for approximating a solution of  $F(x^*) = 0$  for  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is the iteration

$$x_{i+1} = x_i - \alpha_i A_i^{-1} F(x_i) \quad (2.9)$$

Where  $x_i$  represents the  $i$  the approximation to the solution, the matrix  $A_i$  is viewed as an approximation to  $F'(x_i)$  and  $\alpha_i$  called the step length parameter. The choice of  $\alpha_i$  is referred to as step-length control, and we often call the iteration damped if  $0 < \alpha_i$

$\leq 1$ , and refer to  $p_i = -A_i^{-1}F(x_i)$  as the search direction. The success of these iterative methods depend on the choices of  $A_i$  and  $\alpha_i$

Numerical method can be classified into open method and close method, its an iteration method based on the number of previous iterates that the method uses at each iteration. For example, if at the  $i$  step, the iteration depend on precisely  $n$  of the previous iterates  $x_{k-n}, \dots, x_{k-1}$ , this is called the method an  $n$ -point method. An  $n$ -point method requires  $n$  previous iteration (Kelly,1995)

### 2.2.9 Secant method

The secant method is one of the most popular methods for root finding. Standard text books in numerical analysis state that the secant method super linear that used to show the speed of convergence for root-finding.

The secant method is a variation of the Newton-Raphson method due to Newtonself (Kelly,1995). This method requires two initial guesses, but unlike the bisection method, the two initial guesses do not need to bracket the root of the equation.

The secant method is an open method and may or may not converge. However, when secant method converges, it will typically converge faster than the bisection method.



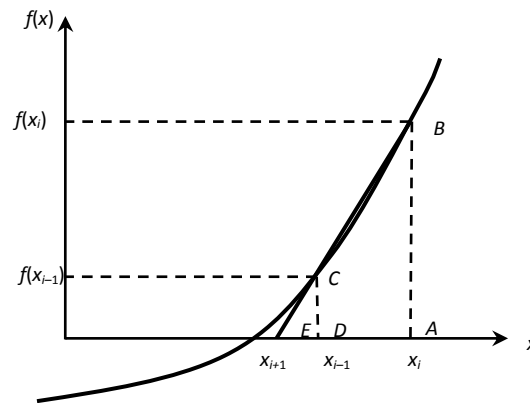


Figure 2.5 Geometrical representation of the secant method.

The secant method can also be derived from geometry, as shown in Figure 2.4. Taking two initial guesses,  $x_{i-1}$  and  $x_i$ , one draws a straight line between  $f(x_i)$  and  $f(x_{i-1})$  passing through the  $x$ -axis at  $x_{i+1}$ .  $ABE$  and  $DCE$  are similar triangles.

Hence

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}} \quad (2.10)$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \quad (2.11)$$

To calculate the absolute relative approximate error  $|\epsilon_a|$

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100 \quad (2.12)$$

Advantages of secant method:

1. It converges faster than a linear rate, so that it is more rapid to convergence than the bisection method.
2. It does not require the use of the derivative of the function, something that is not available in a number of applications.
3. It requires only one function evaluation per iteration, as compared to Newton's method which requires two.

Disadvantages of secant method:

1. It may not converge.
2. There is no error bound guarantee for the computed iteration
3. It is likely to have difficulty if  $f(\alpha) = 0$ . This means the x-axis is tangent to the graph of  $y = f(x)$  at  $x = \alpha$ .
4. Newton's method generalizes more easily to the new methods for solving simultaneous systems of nonlinear equations.
5. Convergence of the secant method is a little slower than Newton's method

### 2.2.10 Convergence

There are many criterion by which can evaluate an iterative procedure, for example the length of time taken to calculate a solution or the amount of computer storage space used in the computation. The convergence rate of a method is as important as a

key measure of performance. Broyden (1970) stated that the rate of convergence of a method is as important as the fact that it converges. There could exist a situation when fast convergence occurs so late that it proves to be less beneficial than slower convergence.

The Secant method converges faster than a method with linear convergence, but slower than a method with quadratic convergence:

$$|X_{f+1}| < \varepsilon \text{ or } \left| \frac{X_{f+1} - X_f}{X_{f+1}} \right| < \delta \quad (2.13)$$

Where  $c_f$  is a constant dependent on the derivative. And this kind of convergence is super-linear



## CHAPTER III

### RESEARCH METHODOLOGY

This chapter explains the research methodology, and the subchapter are as follows: Location and research object, model formulation and development, system characteristic, development inventory model, data collection and processing method, discussion and analysis, and the last is conclusion and recommendation.

#### 3.1 Location and Research Object

This research was conducted in Yopan Ceramics located at Kasongan, Bantul. This research focused on the development of inventory model with variable holding cost to accommodate planned shortage.

#### 3.2 Model Formulation and Development

In this stage, there are two models that used for this research. First, the model from Alfares (2007), that designed to determine the optimal order quantity and cycle time. This model considers the holding cost assumed to be variable and the cost will increase the time spent in storage for longer term. There are two types algorithm models of the variable holding cost based on increasing function of the time spent in storage, namely retroactive increase and incremental increase. For each type, a simple algorithm that minimizes the total inventory cost (TIC) is developed for calculating optimal order quantity and associated cycle time.

In Alfares model's the shortage cost are not allowed. Thus, this research using Shamsi, et.al., (2010) model's as second models that used to develop Alfares model's to accomodate planned shortage.

The model used in this research considers two cost, they are time independent shortage cost and time dependent shortage cost. To accomodate planned shortage must be considered about the administrative cost for determining time independent shortage cost and backorder cost for determining time dependent shortage cost.

### 3.3 System Characteristic

This research was conducted in Ceramic Company. In the production, the company purchased a clay material from the supplier. In this process, the company asked to the supplier to process material that consists of red soil, white soil, and sand in order to be processed to be clay. Then, the company made an order based on the need and inventory on the warehouse, after that the supplier will send to the company. Therefore, the company must pay to place an order and this activity is called the order cost. Once finished, the material will be inspected and then stored in the warehouses, this process caused storage cost (holding cost).

The company production activities are based on the make of order and stock. So, after the availability of materials, the company can execute the production process based on orders from customers and make some stock products for anticipate if there is an additional purchase from other customers. In the process of production, sometimes there is a defect product, so the product will be returned to the production floor to be repaired (rework).

Based on the experience of previous years, companies sometimes have a shortage of materials or products when the demand increases. Therefore, the customer usually will wait until the demand can be fulfilled. Accordingly, the company gets additional cost to fulfill that demand which is usually called a penalty cost (Shortage cost).

### 3.4 Inventory Model Development

#### 3.4.1 Mathematical Notation

$Q(t)$ : the quantity on-hand at time  $t$

$D$  : constant (base) demand rate

$n$  : number of distinct time periods with different holding cost rates

$t$  : time from the start of the cycle at  $t = 0$

$t_i$  : end time of period  $i$ , where  $i=1, 2, \dots, n$ ,  $t_0=0$ , and  $t_n= \infty$

$k$  : ordering cost per order

$h_i$  : holding cost of the item in period  $i$

$h(t)$  : holding cost of the item at time  $t$ ,  $h(t) = h_i$  if  $t_{i-1} \leq t \leq t_i$

$\beta$  : demand parameter indicating elasticity in relation to the inventory level

$B$  : The number of backorders

$\bar{B}$  : Average backorders

$N$  : Number of order

$P$  : Production rate, units per planning period, (units/year).

$\alpha$  : Proportion of defectives in production process in each cycle

$E1$  : Proportion of good items that incorrectly rejected in each cycle

$E2$  : Proportion of bad items that incorrectly accepted in each cycle

$C_s$  : Annual backorder cost per unit short

$C_b$  : Fixed administrative backorder cost per unit stockout

### 3.4.2 Decision Variables

$Q^*$  : optimal order quantity

$T^*$  : cycle time

$TIC$  : Total Inventory Cost

$TC_{\bar{B}}$  : Total time dependent shortage cost

$TC_B$  : Total time independent shortage cost

$TC$  : Total Cost

### 3.4.3 Assumption and Limitations

1. The holding cost is varying as an increasing step function of time in storage.
2. Shortages are allowed
3. A single item is considered
4. Customer is still waiting if the demand cannot be fulfilled and will be backordered by company
5. Error inspection happened in the production process

### 3.4.4 Inventory Model with Variable Holding Cost

This model developed by Alfares (2007), the objective is to minimize Total Inventory Cost (TIC) per unit time, which includes two components: The ordering cost, and the holding cost. Since one order is made per cycle, the ordering cost per unit time is simply  $K/T$ . The total holding cost per cycle is obtained by integrating the product of holding cost  $h(t)$  and inventory level  $q(t)$  over the whole cycle.

$$TIC = \frac{K}{T} + \frac{1}{T} \int_0^T h(t)q(t)dt \quad (3.1)$$

Since the demand rate is equal to the rate of inventory level decrease, it can describe inventory level  $q$  by the following differential equation:

$$\frac{dq(t)}{dt} = -D [q(t)]^\beta, \quad D > 0, 0 \leq t \leq T, 0 < \beta < 1 \quad (3.2)$$



The on-hand inventory level at time  $t$ ,  $q(t)$ , can be evaluated by solving (3):

$$q^{1-\beta} dq = -D dt$$

By integrating both sides

$$\int_0^T q^{1-\beta} dq = \int_0^T -D dt, \text{ where } 0 \leq t \leq T$$

$$\frac{q^{1-\beta}}{(1-\beta)} = -Dt,$$

$$q^{1-\beta}(t) = D(1-\beta)t + q^{1-\beta}(0)$$

However

$$q^{1-\beta}(0) = Q^{1-\beta}$$

Thus,

$$q^{1-\beta}(t) = -D(1-\beta)t + Q^{1-\beta},$$

$$q(t) = [-D(1-\beta)t + Q^{1-\beta}]^{1/(1-\beta)} \quad (3.3)$$

The period  $T$  can be evaluated by substituting the inventory function  $q(t)$  at  $T$ :

$$-q^{1-\beta}(T) = -D(1-\beta)T + Q^{1-\beta} \quad (3.4)$$

Hence,

$$T = \frac{Q^{1-\beta}}{D(1-\beta)} \quad (3.4)$$

Or

$$Q = [D (1-\beta)T]^{1/(1-\beta)} \quad (3.5)$$

### 3.3.5 Optimization

#### A. Case I : Retroactive Holding Cost Increase

As stated earlier, the holding cost is assumed to be an increasing step function of storage time, i.e.  $h_1 < h_2 < \dots < h_n$ . In Case 1, a uniform holding cost that depends on the length of storage is used. Specially, the holding cost of the last storage period is applied retroactively to all previous periods. Thus, if the cycle ends in period  $e$ , ( $t_{e-1} \leq T \leq T_e$ ), then the holding cost rate  $h_e$  is applied to all periods 1, 2, ...,  $e$ . In this case, the TIC per unit time can be expressed as

$$TIC = \frac{K}{T} + \frac{1}{T} \int_0^T h(t) q(t) dt, \quad t_{i-1} \leq T \leq t_e \quad (3.6)$$

Substituting the value of  $q(t)$  from (4)

$$\begin{aligned} TIC &= \frac{K}{T} + \frac{1}{T} \int_0^T -D (1-\beta)t + Q^{1-\beta} J^{1/(1-\beta)} dt, \\ &= \frac{K}{T} - \frac{h_1}{D(2-\beta)T} [-D (1-\beta) + Q^{1-\beta} J^{2-\beta/(1-\beta)}] \end{aligned}$$

Thus,

$$TIC = \frac{K}{T} + \frac{h_1}{D(2-\beta)T} \times [ [Q^{1-\beta} J^{2-\beta/(1-\beta)} - [-D (1-\beta)T + Q^{1-\beta} J^{2-\beta/(1-\beta)}] ]$$

Substituting the value of  $T$  from (3.4)

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{D(2-\beta)T} \quad t_{i-1} \leq T \leq t_i \quad (3.7)$$

Setting the derivative of  $TIC$  with respect to  $Q$  equal to zero and solving for  $Q$ , obtain:

$$Q^* = \left[ \frac{KD(1-\beta)(2-\beta)}{h_i} \right]^{1/(1-\beta)}, \quad Q^* = t_{i-1} \leq T \leq t_i \quad (3.8)$$

### B. Case II: Incremental Holding Cost Increase

According to this function (incremental), higher holding cost rates apply to storage in later periods. Thus, if the cycle ends in period  $e$ , ( $t_{e-1} \leq T \leq T_e$ ), then the holding cost rate  $h_1$  is applied to period 1, rate  $h_2$  is applied to period 2, and so on, therefore rate  $h_e$  is applied only to period  $e$  from time  $t_{e-1}$  up to time  $T$ . For this case, first reset the value of  $t_e$  as ( $t_e = T$ ), and then express the  $TIC$  per unit time as

$$TIC = \frac{K}{T} + \frac{h_1}{T} \int_0^{t_1} q(t) dt + \frac{h_2}{T} \int_{t_1}^{t_2} q(t) dt + \dots + \frac{h_e}{T} \int_{t_{e-1}}^{t_e=T} q(t) dt \quad (3.9)$$

Substituting the value of  $q(t)$  from (4), obtain :

$$\begin{aligned} TIC &= \frac{K}{T} + \sum_{i=1}^e \frac{h_i}{T} \int_{t_{i-1}}^{t_i} [-D(1-\beta)t + Q^{1-\beta}]^{\frac{1}{1-\beta}} dt, \\ &= \frac{K}{T} + \sum_{i=1}^e \frac{h_i}{D(2-\beta)T} [-D(1-\beta)t + Q^{1-\beta}]^{(2-\beta)/(1-\beta)} \end{aligned}$$

Substituting the value of  $T$  from (3.4), rearranging and simplifying terms gives:

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} \quad (3.10)$$

To find the optimal order size  $Q^*$ , we set the derivative of  $TIC$  with respect to  $Q$  equal to zero. After simplification, then obtained:

$$\frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i Q}{(2-\beta)} + \sum_{i=1}^{e-1} (h_{i+1}-h_i)[Q^{1-\beta} - D(1-\beta)t_i]^{1/(1-\beta)} - \sum_{i=1}^{e-1} \frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} x \quad (3.11)$$

$$[Q^{1-\beta} - D(1-\beta)t_i]^{(2-\beta)/(1-\beta)} = 0$$

### 3.3.7 Solution Algorithm Variable Holding Cost

#### A. Case I : Retroactive Holding Cost Increase

1. Start with the lowest holding cost  $h_1$ , use (3.8) to determine  $Q$  and (3.4) to determine  $T$  for each  $i$  until  $Q$  is realizable (i.e.  $t_{i-1} \leq T \leq t_i$ ). Call these values  $T_R$  and  $Q_R$ .

$$Q^* = \left[ \frac{KD(1-\beta)(2-\beta)}{h_1} \right]^{1/(1-\beta)}$$

$$T = \frac{Q^{1-\beta}}{D(1-\beta)}$$

2. Use (3.5) to calculate all break-point values of  $Q$ ,  $t_1 \leq t_i \leq t_R$ , each  $Q_i$  is obtained by substituting  $t_i$  into (3.5).

$$Q = [D(1-\beta)T]^{1/(1-\beta)}$$

3. Use (3.7) to calculate the  $TIC$  for  $Q_R$  and each  $Q_i$  and  $T_C\bar{B}$

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{D(2-\beta)T} + \hat{\pi} \frac{B^2 PL}{2Q(PL-D)}$$

4. Choose the value of  $Q$  that gives the lowest  $TIC$

## B Case II: Incremental Holding Cost Increase

1. Substitute  $h_l$  into (3.8) to determine  $Q_{max}$ , and then substitute  $Q_{max}$  into (3.4) to determine  $T_{max}$ . If  $T_{max} \leq 1$ , stop; the solution  $(Q_{max}, T_{max})$  is optimal.

$$Q^* = \left[ \frac{KD(1-\beta)(2-\beta)}{h_l} \right]^{1/(1-\beta)}$$

$$T = \frac{Q^{1-\beta}}{D(1-\beta)}$$

2. Substitute  $h_n$  into (3.8) to determine  $Q_{min}$ , and then substitute  $Q_{min}$  into (3.4) to determine  $T_{min}$ .

$$Q^* = \left[ \frac{KD(1-\beta)(2-\beta)}{h_l} \right]^{1/(1-\beta)}$$

$$T = \frac{Q^{1-\beta}}{D(1-\beta)}$$

3. Depending on the values of  $T_{min}$  and  $T_{max}$ , determine the possible periods that  $T$  may fall into (i.e., all feasible values of  $e$ ).

4. For each feasible value of  $e$ , solve (3.11) numerically to determine the optimum value of  $Q$ . If  $Q$  corresponds to the correct period, it is considered realizable.

$$-\frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i Q}{(2-\beta)} \sum_{i=1}^{e-1} (h_{i+1} - h_i) [Q^{1-\beta} - D(1-\beta)t_i]^{1/(1-\beta)} - \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)}$$

$$[Q^{1-\beta} - D(1-\beta)t_i]^{(2-\beta)/(1-\beta)} = 0$$

Using secant method to finding root of value of  $Q$

$$Q_{n+1} = Q_n - \frac{f(Q_n) * (Q_n - Q_{n-1})}{f(Q_n) - f(Q_{n-1})}$$

5. Using (3.10), calculate  $TIC$  for each  $Q_R$  and each  $Q_i = Q_{(t)}$  and  $TC_{\bar{B}}$

$$TIC = \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} + \hat{\pi} \frac{B^2 PL}{2Q(PL-D)}$$

6. Choose the value of  $Q$  that gives the lowest  $TIC$

### 3.4.8 Planned Shortage Cost

This model developed by Shamsi, et.al., (2010)

During the normal production time, at rate  $R_1$  which is equal to

$$R_1 = P [(1 - \beta_1)(1 - E_1) + \beta_1 E_2] - D \quad (3.12)$$

For simplification, let  $L = (1 - \beta)(1 - E_1) + \beta E_2$ , Thus we can write  $R$

$$L_1 = PL - D$$

The required processing time for this quantity (the production time),  $t_p$ , is equal to  $Q_1/P$ , Includes  $t_1$  and  $t_2$  respectively

$$t_1 = \frac{B}{R_1} \quad (3.13)$$

$$t_2 = t_p - t_1 \quad (3.14)$$

Cycle time,  $T$ , includes regular production time, rework process time and the idle time, i.e.,

$$T = t_p + t_r + t_d$$

$t_d$  includes  $t_3$  and  $t_4$  where  $t_3 = T - t_4 - t_r - t_p$ ,  $t_4 = B/D$  and

$$T = \frac{Q}{D} \quad (3.15)$$

Shortage costs include the time dependent costs, the average backorder ( $\bar{B}$ ), can be computed as

$$\bar{B} = \frac{B(t_1 + t_4)}{2T} \quad (3.16)$$

Substituting eqs. (3.12) – (3.13) and (3.15) into Eq. (3.16), obtained

$$\bar{B} = \frac{B^2 PL}{2Q(PL-D)} \quad (3.17)$$

### 3.4.9 Solution Algorithm Planned Shortage Cost

#### A. Time-dependent shortage costs

- 1 Determine average backorder ( $\bar{B}$ )

$$\bar{B} = \frac{B^2 PL}{2Q(PL-D)}$$

- 2 Calculate total time dependent shortage cost ( $TC_{\bar{B}}$ )

$$TC_{\bar{B}} = C_s \times \bar{B} \quad (3.18)$$

#### B Time independent shortage costs

1. Determine annual number of orders

$$N = \frac{D}{Q} \quad (3.19)$$

- 2 Calculate total cost backorders

$$TC_B = NxC_b \times B \quad (3.20)$$

### 3.5 Collecting and Processing Data

Collecting and processing data are conducted to know the built which is mathematical modelling. The mathematical modelling consist of retroactive holding increase algorithm, incremental holding increase algorithm, and planned shortage.

### 3.6 Discussion and Result Analysis

This step is conducted to analyze the data and discuss the result. It is supposed to know what the strength and weakness of this system. Therefore, this research could be used as references to company to policies that related to nventory

### 3.7 Conclusion and Suggestion

This last step is the answer of problem formulation for this research based on processing data and analysis. This section also comes with suggestions for improvements to the research.

### 3.8 Research Flow Diagram

The research steps are required to be organized properly in order to simplify the composing of research report. The following Figure 3.1 is the presentation of the research steps:



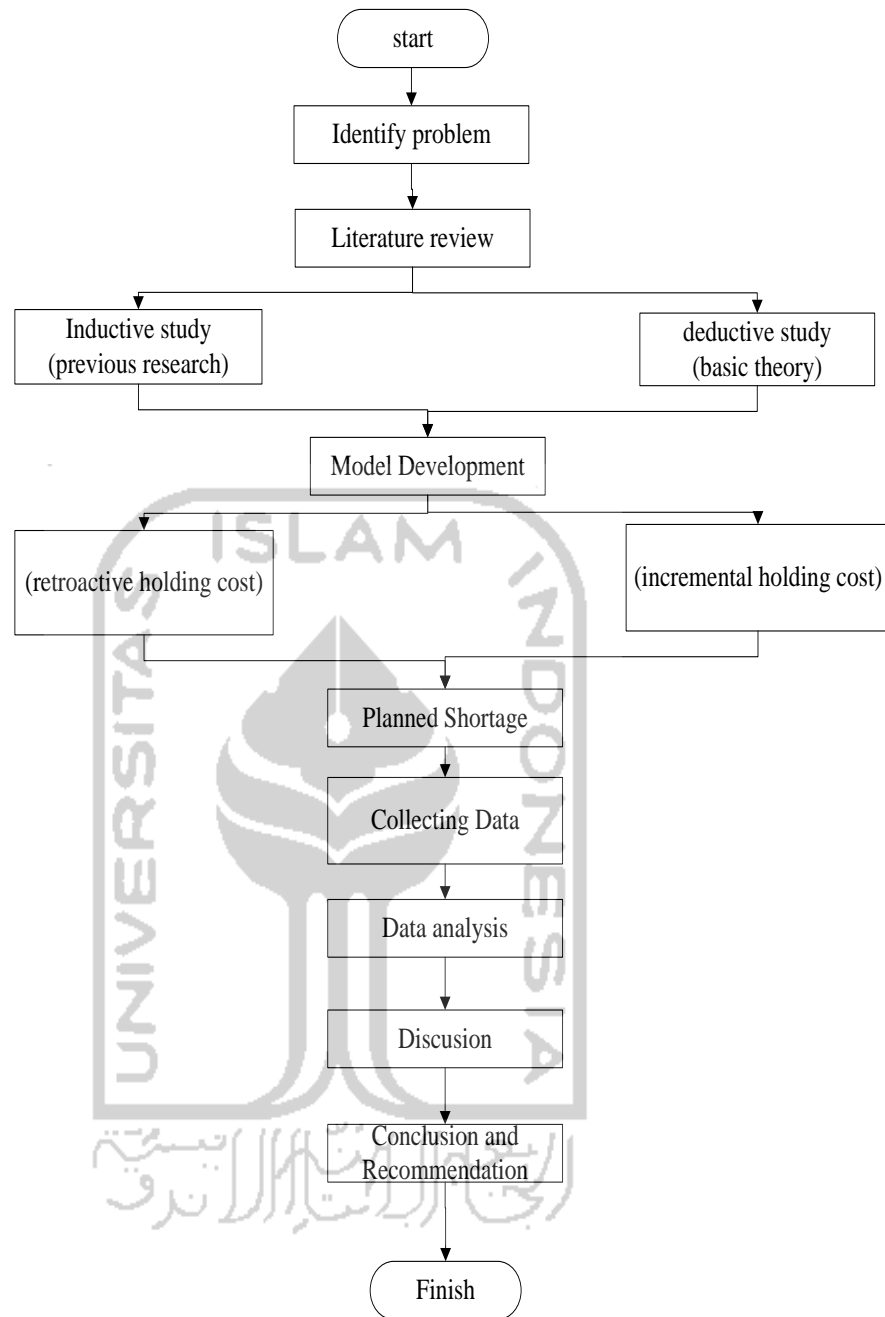


Figure 3.1 Research Framework

## CHAPTER IV

### DATA COLLECTING AND PROCESSING

#### 4.1 Data Collecting

The data collecting was conducted at Yopan ceramic, Kasongan Bantul. The product of yopan ceramic is spread ed out at java area such as Blitar, Surabaya, Jakarta and foreign country, such as Netherland, and U.S America.

##### 4.1.1 Demand

Table 4.1 Actual demand

<b>Periode</b>	<b>demand</b>
October 2010	213
November 2010	209
December 2010	297
Januari 2011	213
Febuari 2011	314
Maret 2011	196
April 2011	294
Mei 2011	287
Juni 2011	217
Juli 2011	212
Agustus 2011	281
September 2011	292

#### 4.1.2 Production rate

Table 4.2 Production rate

Periode	Production
October 2010	266
November 2010	275
December 2010	282
Januari 2011	269
Febuari 2011	295
Maret 2011	253
April 2011	283
Mei 2011	269
Juni 2011	283
Juli 2011	287
Agustus 2011	263
September 2011	275

#### 4.1.3 Order Cost

- a. Telephone time = 20 minutes
- b. Telephone cost =Rp.400,-/minutes
- c. Processing order = Rp. 250.000
- d. Expedition cost = Rp. 50.0000

#### 4.1.4 Holding Cost

- |                           |                     |
|---------------------------|---------------------|
| a. Electricity cost       | = Rp. 100.000/month |
| b. Warehousing staff cost | =Rp. 120.000/month  |
| c. Warehousing staff      | = 2 staff           |

#### 4.1.5 Shortage Cost

This shortage cost can be divided by administrative backorder cost and backorder cost. The administrative estimated Rp.500/unit order and backorder cost estimated Rp. 2500/unit

#### 4.1.6 Defect item and Error Inspection (units)

A. defective items in production

Defective items are caused by the imperfect burning process so the product is cracked or broken

B. Error Inspection

Error Inspection is caused by incorrect inspection on products quality by the worker.

So, several products could be incorrectly rejected or accepted. Item that is incorrectly inspected is occurred when the item is moving to the finishing process.

Reject items which are incorrectly accepted will be reworked. While, good item that is incorrectly rejected will be stored. Since the second condition is rarely occurred in Yopan ceramics. This research assumes that the value of good item which is incorrectly rejected is independently determined.

Table 4.3 Defect item and Error Inspection

Defect	Error inspection	
	rejected item (incorretly accepted)	Good item (incorretly rejected)
50 units	7 units	3 units

## 4.2 Data Processing

### 4.2.1 Total Demand in year ( $D$ )

Table 4.4 Total demand

Periods	demand
October 2010	213
November 2010	209
December 2010	297
Januari 2011	213
Febuari 2011	314
Maret 2011	196
April 2011	294
Mei 2011	287
Juni 2011	217
Juli 2011	212
Agustus 2011	281
September 2011	292
TOTAL	3025

So, the total demand in year 3025unit/year

### 4.2.2 Total Production rate in year( $P$ )

Table 4.5 Total production rate

Periode	Production
October 2010	266
November 2010	275
December 2010	282
Januari 2011	269
Febuari 2011	295
Maret 2011	253
April 2011	283
Mei 2011	269
Juni 2011	283
Juli 2011	287
Agustus 2011	263
September 2011	275
TOTAL	3300

So, the total production rate in year 3300 unit/year

#### 4.2.3 Total order cost per order ( $k$ )

Telephone time = 20 minutes

Telephone cost = Rp. 400, /minutes

Processing order = Rp. 250.000

Expedition cost = Rp. 50.0000

Total cost per order :

$$= (20 \times 400) + 250000 + 50000$$

$$= 8000 + 250000 + 50.000$$

$$= \text{Rp } 308.000/\text{order}$$

#### 4.2.4 Demand parameter elasticity in relation to inventory ( $\beta$ )

Total inventory in year = total production rate ( $P$ ) – Total Demand ( $D$ )

$$= 3300 - 3025$$

$$= 275 \text{ units}$$

So, the demand parameter elasticity to inventory

$$\beta = \frac{\text{Inventory}}{\text{Demand}}$$

$$= \frac{275}{3025}$$

$$= 0,0909 \sim 0,1$$

So, the demand parameter elasticity to inventory is 0,1

#### 4.2.5 Holding cost per unit per year ( $h$ )

1. Electricity cost

$$= \text{Rp. } 100.000/\text{month} \times 12 \text{ month/year}$$

$$= \text{Rp. } 1.200.000,-/\text{year}$$

2. Warehouse staff cost

$$= \text{Rp. } 120.000/\text{month} \times 2 \text{ persons} \times 12$$

$$= \text{Rp. } 2.880.000,-/\text{tahun}$$

$$\text{Total Holding cost in year} = \text{Rp. } 1.200.000 + \text{Rp. } 2.880.000$$

$$= \text{Rp. } 4.080.000,-/\text{year}$$

So the total holding per unit per year :

$$= \frac{\text{Total holding cost in year}}{\text{Production rate}}$$

$$= \frac{\text{Rp. } 4.080.000}{3300 \text{ unit/year}} = \text{Rp. } 1236,364/\text{unit/year}$$

So the total holding costs per unit per year can be estimated Rp1200/units/year

Based on this research, the holding cost is an increasing of the time spent in storage ( $t_{e-1} \leq T \leq t_e$ ) so,

Table 4.6 Variable holding cost

Holding cost	Range time	Cycl time
$h_1 = 400$	$0 \leq T \leq 0,2$	$t_1 = 0,2$
$h_2 = 800$	$0,2 \leq T \leq 0,4$	$t_2 = 0,4$
$h_3 = 1200$	$0,4 \leq T \leq \infty$	$t_3 = \infty$

#### 4.2.6. Proportion of defect and error inspection (units)



1. Proportion of defect ( $\alpha$ )

$$= \frac{\text{Total defect (units/year)}}{\text{Production rate}}$$

$$= \frac{50 \text{ units/year}}{3300 \text{ unit/year}}$$

$$= 0,015 \text{ units/year}$$

## 2. Proportion of reject item (incorrely accepted) (E2)

$$= \frac{\text{Total reject item}}{\text{Production rate}}$$

$$= \frac{7 \text{ units/year}}{3300 \text{ unit/year}}$$

$$= 0,002 \text{ units/year}$$

## 3. Proportion of Good item (incorrely rejected) (E1)

$$= \frac{\text{Total good item}}{\text{Production rate}}$$

$$= \frac{3 \text{ units/year}}{3300 \text{ unit/year}}$$

$$= 0,0009 \text{ units/year}$$



### 4.3 Retroactive Holding Cost Increase

In retroactive holding cost increase, a uniform holding cost depends on the length of storage used. Then, the holding cost of the last storage period is retroactively applied. Thus, if the cycle period ends in period  $e$  ( $t_{e-1} \leq T \leq t_e$ ), then the holding cost rate  $h_e$  is applied to all periods  $1, 2, \dots, e$

The steps of retroactive holding cost increase calculation are :

**Step1.**  $Q$  and  $T$  value is determined from the lowest holding cost  $h_1$  until  $Q$  value is realizable. Using equation (3.8) to determine  $Q$  and use equation (3.4) to determine  $T$

### Iteration 1

$$h_1 = \text{Rp.}400$$

$$\begin{aligned}
 Q^* &= \left[ \frac{KD(1-\beta)(2-\beta)}{h_1} \right]^{(1/(1-\beta))} \\
 &= \left[ \frac{308.000 * 3025(1-0,1)(2-0,1)}{400} \right]^{(1/(1-0,1))} \\
 &= 2977,177 \sim 2978 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 T^* &= \frac{Q^{1-\beta}}{D(1-\beta)} \\
 &= \frac{2978^{1-0,1}}{3025(1-0,1)} \\
 &= \frac{1338,2345}{2722,5}
 \end{aligned}$$

$$= 0,491 \text{ years (unrealizable)}$$

### Iteration 2

$$h_2 = \text{Rp.}800$$

$$\begin{aligned}
 Q^* &= \left[ \frac{KD(1-\beta)(2-\beta)}{h_2} \right]^{1/(1-\beta)} \\
 &= \left[ \frac{308.000 \cdot 3025(1-0,1)(2-0,1)}{800} \right]^{1/(1-0,1)} \\
 &= 2067,088 \sim 2068 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 T^* &= \frac{Q^{1-\beta}}{D(1-\beta)} \\
 &= \frac{2068^{1-0,1}}{3025(1-0,1)} \\
 &= 0,35402 \sim 0,35 \text{ years (realizable)}
 \end{aligned}$$

### Iteration 3

$$h_3 = \text{Rp.}1200$$

$$\begin{aligned}
 Q &= \left[ \frac{KD(1-\beta)(2-\beta)}{h_i} \right]^{1/(1-\beta)} \\
 &= \left[ \frac{308000 \cdot 3025(1-0,1)(2-0,1)}{1200} \right]^{1/(1-0,1)} \\
 &= 1669,858 \sim 1670 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 T &= \frac{Q^{1-\beta}}{D(1-\beta)} \\
 &= \frac{1670^{1-0,1}}{3025(1-0,1)}
 \end{aligned}$$

$$= 0,292 \text{ year (not realizable)}$$

**Step 2.** Calculate all break-point values of  $Q$  using equation (3.5)

**Iteration 1**

$$\begin{aligned} Q_1 &= [D(1-\beta)T]^{1/(1-\beta)} \\ &= [3025(1-0,1)0,2]^{1/(1-0,1)} \\ &= 1096,472 \sim 1097 \text{ units} \end{aligned}$$

**Iteration 2**

$$\begin{aligned} Q_2 &= [D(1-\beta)T]^{1/(1-\beta)} \\ &= [3025(1-0,1)0,4]^{1/(1-0,1)} \\ &= 2368,511 \sim 2369 \text{ units} \end{aligned}$$

**Step 3.** Calculate the total inventory cost ( $TIC$ ) for  $Q_R$  and each  $Q_i$  using equation (3.7)

**Iteration 1**

$$\begin{aligned} TIC(2068) &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} \\ &= \frac{308000 \cdot 3025(1-0,1)}{2068^{1-0,1}} + \frac{800(1-0,1)2068}{(2-0,1)} \\ &= 870007,75 + 783663,15 \\ &= \text{Rp.1.653.670,90/year} \end{aligned}$$

**Iteration 2**

$$\begin{aligned}
 TIC (1097) &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i (1-\beta)Q}{(2-\beta)} \\
 &= \frac{308000*3025(1-0,1)}{1097^{1-0,1}} + \frac{400 (1-0,1)1097}{(2-0,1)} \\
 &= 1539333,21 + 207852,63 \\
 &= \text{Rp.1.747.185,8/year}
 \end{aligned}$$

**Iteration 3**

$$\begin{aligned}
 TIC (2369) &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i (1-\beta)Q}{(2-\beta)} \\
 &= \frac{308000*3025(1-0,1)}{2369^{1-0,1}} + \frac{800(1-0,1)2369}{(2-0,1)} \\
 &= 769856,9775 + 897726,3158 \\
 &= \text{Rp.1.667.583,293/year}
 \end{aligned}$$

**Step 4.** Choose the value of Q that gives the lowest TIC

Based the iteration above, the optimum solution obtained is  $Q = 2068$  units,  $T = 0,35$  years, and  $TIC = \text{Rp. 1.653.670,90 /year}$

#### 4.4 Incremental Holding Cost Increase

In Incremental holding increase, higher storage cost rates is applied to storage in later periods. Thus, if the cycle ends in period  $e$  ( $t_{e-1} \leq T \leq t_e$ ), the holding cost rate  $h_1$  is applied to period 1, rate  $h_2$  is applied to period 2 then  $h_e$  is applied to period  $e$

**Step1.** Determine  $Q_{max}$  using equation (3.8) and  $T_{max}$  use equation(3.4)

From previous calculation (retroactive holding increase),  $Q_{max}$  and  $T_{max}$  is obtained:  $Q_{max} = 2978$  units and  $T_{max} = 0,491$  years. Since  $T_{max} > t_1$ , the process is continued

**Step 2.** Determine  $Q_{min}$  using equation (3.8) and  $T_{min}$  use equation(3.4)

$$\begin{aligned}
 Q_{min} &= \left[ \frac{KD(1-\beta)(2-\beta)}{h_1} \right]^{1/(1-\beta)} \\
 &= \left[ \frac{308000 * 3025(1-0.1)(2-0.1)}{1200} \right]^{1/(1-0.1)} \\
 &= 1669,858 \sim 1670 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 T_{min} &= \frac{Q^{1-\beta}}{D(1-\beta)} \\
 &= \frac{1670^{1-0.1}}{3025(1-0.1)} \\
 &= 0,292 \text{ years}
 \end{aligned}$$

**Step 3.** Determinine the possible periods of  $T$ , Depending on the values of  $T_{min}$  and  $T_{max}$

Since  $T_{min}$  included period 2 and  $T_{max}$  included period 3. The total inventory cost to be calculated only for period 2 and 3

**Step 4.** Determine the optimum value of  $Q$  using equation (3.11) numerically for each feasible value of  $e$

a)  $e=2$ , with  $T$  and  $Q$  range is  $0, 292 \leq T \leq 0,4$  and  $1670 \leq Q \leq 2369$

$$-\frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i Q}{(2-\beta)} + \sum_{i=1}^{e-1} (h_{i+1} - h_i) [Q^{1-\beta} - D(1-\beta)t_i]^{\frac{1}{1-\beta}} - \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times$$

$$[Q^{1-\beta} - D(1-\beta)t_i]^{\frac{2-\beta}{1-\beta}} = 0$$

$$-\frac{308000 \cdot 3025(1-0,1)}{Q^{1-0,1}} + \frac{400 Q}{(2-0,1)} + \sum_{i=1}^{e-1} (800-400) [Q^{1-0,1} - 3025(1-0,1)0,2]^{\frac{1}{1-0,1}} -$$

$$\sum_{i=1}^{e-1} \frac{(800-400)(1-0,1)}{Q^{1-0,1}(2-0,1)} \times [Q^{1-0,1} - 3025(1-0,1)0,2]^{\frac{2-0,1}{1-0,1}} = 0$$

So the equation is

$$-\frac{838530000}{Q^{0,9}} + \frac{400 Q}{1,9} + 400 [Q^{0,9} - 544,5]^{\frac{1}{0,9}} - \frac{360}{1,9Q^{0,9}} \times [Q^{0,9} - 544,5]^{1,9/0,9} = 0$$

Solve this equation by the secant method, using range limit  $Q=1670$  and  $Q=2369$  as initial values, So

$$Q_1 = 1670 \text{ units} \quad f(Q_1) = -545427$$

$$Q_2 = 2369 \text{ units} \quad f(Q_2) = 63711,33275$$

Secant method :

$$Q_{n+1} = Q_n - \frac{f(Q_n) * (Q_n - Q_{n-1})}{f(Q_n) - f(Q_{n-1})}$$

**Iteration 1**

$$Q_{(3)} = Q_2 - \frac{f(Q_2) * (Q_2 - Q_1)}{f(Q_2) - f(Q_1)}$$

$$= 2369 - \frac{63711,33275 * (2369 - 1670)}{63711,33275 - (-545427)}$$

$$= 2295,889837 \text{ units}$$

Absolute relative approximate error  $|\epsilon_a|$  for iteration 1

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$|\epsilon_a| = \left| \frac{2369 - 1670}{2369} \right| \times 100$$

$$= 29,506 \%$$

**Iteration 2**

$$Q_{(4)} = Q_3 - \frac{f(Q_3) * (Q_3 - Q_2)}{f(Q_3) - f(Q_2)}$$

$$= 2295,889837 - \frac{8450,587216 * (2295,889837 - 2369)}{8450,587216 - 63711,33}$$

$$= 2284,70968 \text{ units}$$



Absolute relative approximate error  $|\epsilon_a|$  for iteration 2

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$|\epsilon_a| = \left| \frac{2295,88 - 2369}{2295,88} \right| \times 100$$

$$= 3,184 \%$$

### Iteration 3

$$Q_{(5)} = Q_4 - \frac{f(Q_4) * (Q_4 - Q_3)}{f(Q_4) - f(Q_3)}$$

$$= 2284,70968 - \frac{(-129,455102) * (2284,70968 - 2295,889837)}{(-129,455102) - 8450,587216}$$

$$= 2284,878366 \text{ units}$$

Absolute relative approximate error  $|\epsilon_a|$  for iteration 3

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$|\epsilon_a| = \left| \frac{2284,709 - 2295,88}{2284,709} \right| \times 100$$

$$= 0,489 \%$$

### Iteration 4

$$Q_{(6)} = Q_5 - \frac{f(Q_5) * (Q_5 - Q_4)}{f(Q_5) - f(Q_4)}$$

$$= 2284,878366 - \frac{0,264100873 * (2284,878366 - 2284,70968)}{0,264100873 - (-129,456)}$$

$$= 2284,878023 \text{ units}$$

Absolute relative approximate error  $|\epsilon_a|$  for iteration 4

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$|\epsilon_a| = \left| \frac{2284,878 - 2284,709}{2284,878} \right| \times 100$$

$$= 0,00738 \%$$

**Iteration 5**

$$Q_{(7)} = Q_6 - \frac{f(Q_6) * (Q_6 - Q_5)}{f(Q_6) - f(Q_5)}$$

$$= 2284,878023 - \frac{8,2578 \cdot 10^{-6} * (2284,878023 - 2284,878366)}{8,2578 \cdot 10^{-6} - 0,264100873}$$

$$= 2284,878023 \text{ units}$$

Absolute relative approximate error  $|\epsilon_a|$  for iteration 5

$$|\epsilon_a| = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$|\epsilon_a| = \left| \frac{2284,8783 - 2284,8780}{2284,878} \right| \times 100$$

$$= 0,00002 \%$$

Value  $Q$  already convergence, so  $Q = 2285$  units (realizable, since  $e = 2$  means  $1097 \leq Q \leq 2369$ , thus  $Q_R = 2285$ )

b)  $e=3$ , with  $T$  and  $Q$  range is  $0,4 \leq T \leq 0,491$  and  $2369 \leq Q \leq 2978$

$$-\frac{308000 \cdot 3025(1-0,1)}{Q^{1-0,1}} + \frac{400 Q}{(2-0,1)} + (800-400)[Q^{1-0,1} - 3025(1-0,1)0,2]^{1-0,1} - \frac{1}{Q^{1-0,1}(2-0,1)} [Q^{1-0,1} - 3025(1-0,1)0,2]^{2-0,1} + (1200-800)[Q^{1-0,1} - 3025(1-0,1)0,4]^{1-0,1} - \frac{(1200-800)(1-0,1)}{Q^{1-0,1}(2-0,1)} x [Q^{1-0,1} - 3025(1-0,1)0,24] = 0$$

So the equation is

$$-\frac{838530000}{Q^{0,9}} + \frac{400 Q}{1,9} + 400[Q^{0,9} - 544,5]^{0,9} - \frac{360}{1,9Q^{0,9}} x [Q^{0,9} - 544,5]^{1,9} + 400[Q^{0,9} - 1089]^{0,9} - \frac{360}{1,9Q^{0,9}} x [Q^{0,9} - 1089]^{1,9/0,9} = 0$$

Solve this equation by the secant method, using range limit  $Q=2369$  and  $Q=2978$  as initial values. So

$$Q_1 = 2369 \text{ units } f(Q_1) = 63779,08683$$

$$Q_2 = 2978 \text{ units } f(Q_2) = 647554,2756$$

Secant method :

$$Q_{n+1} = Q_n - \frac{f(Q_n) \cdot (Q_n - Q_{n-1})}{f(Q_n) - f(Q_{n-1})}$$

**Iteration 1**

$$\begin{aligned}
 Q_{(3)} &= Q_2 - \frac{f(Q_2) \cdot (Q_2 - Q_1)}{f(Q_2) - f(Q_1)} \\
 &= 2978 - \frac{647554,2756 \cdot (2978 - 2369)}{647554,2756 - 63779,08683} \\
 &= 2302,465031 \text{ units}
 \end{aligned}$$

Absolute relative approximate error  $|\epsilon_a|$  for iteration 1

$$\begin{aligned}
 |\epsilon_a| &= \left| \frac{x_1 - x_0}{x_1} \right| \times 100 \\
 |\epsilon_a| &= \left| \frac{2978 - 2369}{2978} \right| \times 100 \\
 &= 20,449\%
 \end{aligned}$$

Based on calculation, iteration 2 is divergence, so  $Q = 2302$  is not realizable

**Step 5.** Calculate the total inventory cost (*TIC*) using equation (18) for each  $Q_R$  and each  $Q_i$

### Iteration 1

$$\begin{aligned}
 TIC(1097) &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} \\
 &= \frac{308000 \cdot 3025(1-0,1)}{Q^{0,9}} + \frac{400(1-0,1)Q}{(2-0,1)} + \frac{(800-400)(1-0,1)}{Q^{1-0,1}(2-0,1)} \times
 \end{aligned}$$

$$\begin{aligned}
& [Q^{1-0,1} - 3025 (2-0,1)0,2]^{\frac{2-0,1}{1-0,1}} \\
&= \frac{838530000}{Q^{0,9}} + \frac{360 Q}{1,9} + \frac{360}{1,9Q^{0,9}} \times [Q^{0,9} - 544,5]^{1,9/0,9} \\
&= 1539333,214 + 207852,6316 + 0,00087 \times 0,04738 \\
&= \text{Rp.1.747.185,8}
\end{aligned}$$

### Iteration 2

$$\begin{aligned}
\text{TIC (2285)} &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} \\
&= \frac{308000 \cdot 3025(1-0,1)}{Q^{0,9}} + \frac{400(1-0,1)Q}{(2-0,1)} + \frac{(800-400)(1-0,1)}{Q^{1-0,1}(2-0,1)} \times \\
& [Q^{1-0,1} - 3025 (2-0,1)0,2]^{(2-0,1)/(1-0,1)} \\
&= \frac{838530000}{Q^{0,9}} + \frac{360 Q}{1,9} + \frac{360}{1,9Q^{0,9}} \times [Q^{0,9} - 544,5]^{1,9/0,9} \\
&= 795281,7583 + 432947,3684 + 0,000449 \times 519717,79 \\
&= \text{Rp.1.228.462.61}
\end{aligned}$$

### Iteration 3

$$\begin{aligned}
\text{TIC (2369)} &= \frac{KD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)} + \sum_{i=1}^{e-1} \frac{(h_{i+1}-h_i)(1-\beta)}{Q^{1-\beta}(2-\beta)} \times [Q^{1-\beta} - D(2-\beta)t_i]^{(2-\beta)/(1-\beta)} \\
&= \frac{308000 \cdot 3025(1-0,1)}{Q^{0,9}} + \frac{400(1-0,1)Q}{(2-0,1)} + \frac{(800-400)(1-0,1)}{Q^{1-0,1}(2-0,1)} \times
\end{aligned}$$

$$\begin{aligned}
& [Q^{1-0,1} - 3025 (2-0,1)0,2]^{(2-0,1)/(1-0,1)} \\
&= \frac{838530000}{Q^{0,9}} + \frac{360 Q}{1,9} + \frac{360}{1,9Q^{0,9}} \times [Q^{0,9} - 544,5]^{1,9/0,9} \\
&= 769856,9775 + 448863,1579 + 0,000435 \times 597497,55 \\
&= \text{Rp.1.218.979,98}
\end{aligned}$$

**Step 6.** Choose the value of  $Q$  that gives the lowest  $TIC$

Based the iteration above, the optimum solution obtained is  $Q = 2369$ ,  $T^* = 0,4$  And  $TIC = \text{Rp.1.218.979,98/year}$

The result of comparasion between retroactive and incremental holding cost increase shows that the total inventory cost of incremental holding cost less costly than retroactive holding cost.

#### 4.5 Planned Shortage

To planned shortage, consider the optimum backorder ( $B^*$ ) based on optimal order size ( $Q^*$ ) that already be calculated. To calculate optimum backorder using equation (3.18)

$$\begin{aligned}
B^* &= \frac{h Q^* - DC_b}{h + C_s} \\
&= \frac{(1200 \times 2369) - (3025 \times 500)}{1200 + 2500} \\
&= \frac{2842800 - 1512500}{1200 + 2500}
\end{aligned}$$

$$= 359,540 \sim 360 \text{ units}$$

In planned shortage models there can be both time-dependent and time-independent shortage costs

**a. Time-dependent shortage costs**

**Step 1. Determine average backorder ( $\bar{B}$ )**

- (i) Calculate the normal production time at rate  $R_1$  using equation (3.12)

$$\begin{aligned} R_1 &= P [(1-\alpha)(1-E_1) + \alpha E_2] - D \\ &= 3300 [(1-0,015)(1-0,002) + 0,015 * 0,0009] - 3025 \\ &= 3244,044 - 3025 \\ &= 219,0436 \sim 220 \text{ units} \end{aligned}$$

- (ii) Determine average backorder using equation (3.17). For simplification, let  $L = (1-\beta)(1-E_1) + \beta E_2$ ,

$$\bar{B} = \frac{B^2 PL}{2Q (PL-D)}$$

$$= \frac{360^2 \times 3244,044}{2(2369) \times 220}$$

$$= 403,342 \sim 404 \text{ units}$$

**Step 2.** Calculate total time dependent shortage cost ( $TC_{\bar{B}}$ ) using equation (3.18)

$$\begin{aligned} TC_{\bar{B}} &= C_s \times \bar{B} \\ &= 2500 \times 404 \\ &= \text{Rp.}1.010.000/\text{year} \end{aligned}$$

**b. Time independent shortage costs**

**Step 1** Determine annual number of orders using equation (3.19)

$$\begin{aligned} N &= \frac{D}{Q} \\ &= \frac{3025}{2369} \\ &= 1,27691 \sim 1 \text{ orders} \end{aligned}$$

**Step 2** Calculate total cost backorders using equation (3.20)

$$\begin{aligned} TC_B &= N_o \times C_b \times B \\ &= 1 \times 500 \times 360 \\ &= \text{Rp.} 180.000 \end{aligned}$$

#### 4.6 Total annual cost

$$TC = TIC + (TC_{\bar{B}} + TC_B)$$



$$= \text{Rp.1.218.979} + (\text{Rp.1.010.000} + \text{Rp. 180.000})$$

$$= \text{Rp.1.218.979} + \text{Rp.1.170.000}$$

$$= \text{Rp. 2.408.979}$$



## CHAPTER V

### DISCUSSION

#### 5.1 Model Analysis

Based on the processing data in chapter IV, there are some data that shows that the demand greater than production rate and this condition creates a shortage. In the figure 5.1, the shortage condition happen at CV. Yopan Ceramics on the December 2010, March 2011, April 2011, May 2011, August 2011, and September 2011.

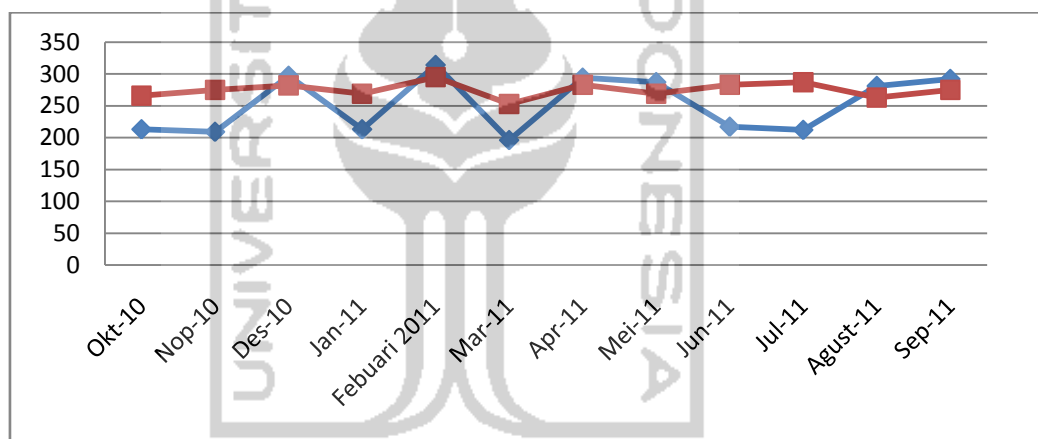
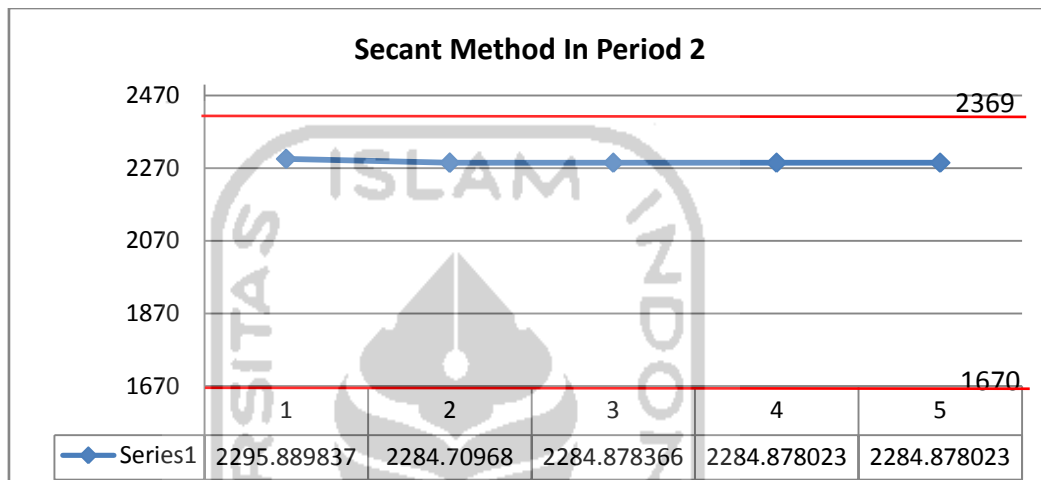


Figure 5.1 Production and Demand graph

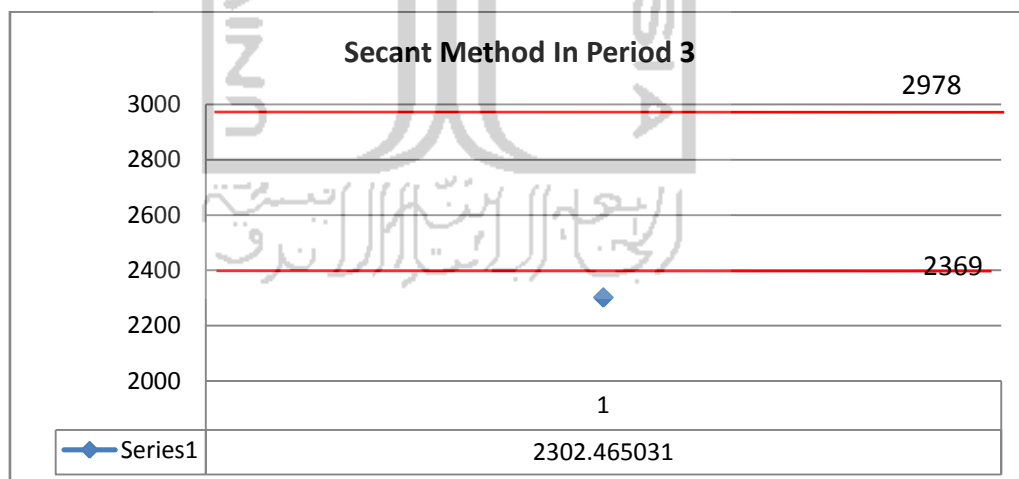
As explanation in previous chapter, based on the real condition at CV, Yopan Ceramics related with has inventories in a storage. Therefore, to store the inventories need costs that called holding cost, and value of holding cost is variable based on time spent in storage. To solve that problem, this research using mathematical way by Alfares model's to represent optimal solution in variable holding cost. There are two type of variable holding cost that are retroactive and incremental holding cost and also divided into 3 different periods, that are short term, medium term, and long

term. Firstly, the final result of retroactive holding cost shows  $Q^* = 2068$ ,  $T^* = 0,35$  and  $TIC^* = \text{Rp.}1.667.583,29$ .

Based on the function of incremental holding cost that the unit stored only used for particular periods, thus the result only calculated in period 2 and 3 using secant method that need two initial guess ( $Q_1$  and  $Q_2$ ) to find the optimum value



**Figure 5.2** Convergence of Secant Method for period 2



**Figure 5.3** divergence of Secant Method for period 3

The convergence graph above shows that period 2 is convergence at fifth iterations and also still in control of the range of period 2 but in period 3 the value is out of control. Thus, its indicate that the value of period 2 is realizable and the result of incremental holding cost increase shows  $Q^* = 2369$ ,  $T^* = 0,4$

and  $TIC^* = \text{Rp.}1.218.979,98/\text{years}$ . Based on the result of TIC shows that the incremental holding cost lower than retroactive Therefore, incremental function in holding cost less costly and to be the best optimum solution than retroactive function.

To answer the real condition problem that the shortages can't be avoided because of uncertainties such as the demand greater than availability and material availability. Thus, company need to accomodate planned shortage to anticipate that conditions and the customer demand could be fulfilled quickly. Based on Shamsi ,et.al., model's, in planned shortage there can be both time that are time dependent that consider the backorder cost and time independent that related with administrative backorder cost. The result shows the  $B^* = 360$  units,  $TC_{\bar{B}} = \text{Rp.}1.110.000$ , and  $TC_B = \text{Rp.}180.000$

## 5.2 Sensitivity Analysis

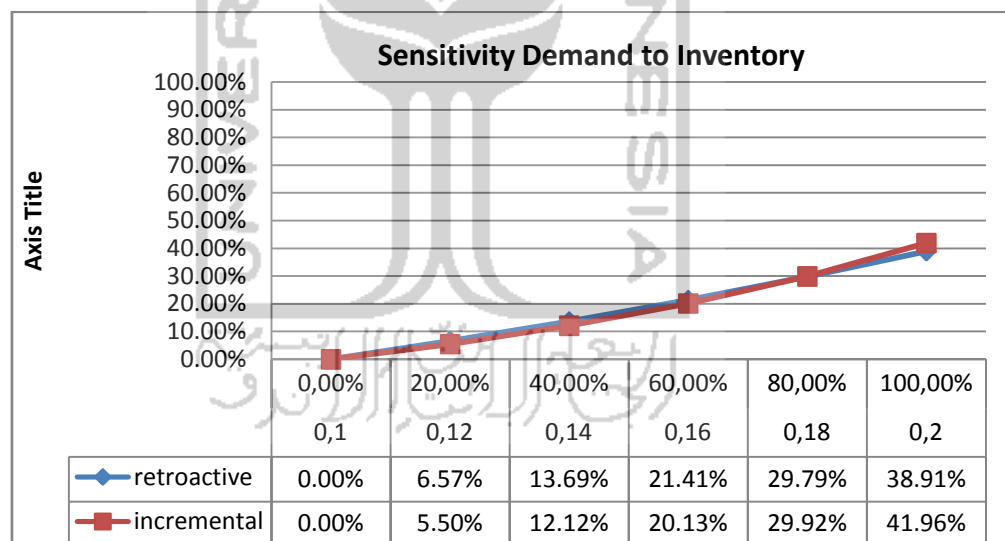
Sensitivity analysis is used to see how much the changes of Total Inventory Cost (TIC) by changing the parameter. The parameter that will be used is the demand elasticity to inventory ( $\beta$ ), holding cost ( $h$ ), and period time ( $t$ ). The sensitivity analysis as follow;

### 5.2.1 Changes of Demand Elasticity to Inventory( $\beta$ )

The sensitivity analysis for demand elasticity to inventory as follows;

In figure 5.4 shows the sensitivity of demand elasticity to inventory, then the effects  $TIC$  over the demand elasticity to inventory ( $\beta$ ) are studied by changing the  $\beta$  values over the range from 0.00% to 100%. If the demand elasticity increase 20%, it will be increasing in  $TIC$  to 6,57% for retroactive holding cost, and 5,50% for incremental holding cost. If the demand elasticity increase 40%, it will be increasing in  $TIC$  to

13,69% for retroactive holding cost, and 12,12% for incremental holding cost. If the demand elasticity increase 60%, it will be increasing in *TIC* to 21,41% for retroactive holding cost, and 20,13% for incremental holding cost. If the demand elasticity increase 80%, it will be increasing in *TIC* to 29,79% for retroactive holding cost, and 29,92% for incremental holding cost. If the demand elasticity increase 100%, it will be increasing in *TIC* to 38,91% for retroactive holding cost, and 41,96% for incremental holding cost. It is observed that the total inventory cost are directly related with demand elasticity to inventory and its effect becomes increasing inventory cost if the value of demand elasticity to be increased. Based on the figure 5.4, can be analyzed that the total inventory cost using incremental model higher than using retroactive if the demand elasticity changed to 80%



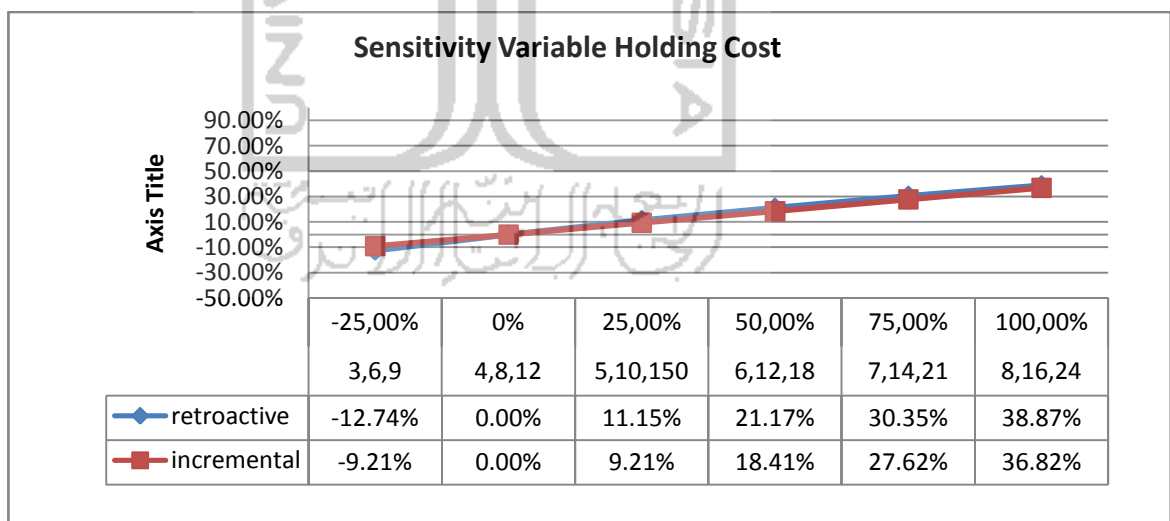
**Figure 5.4**Changes of Demand elasticity to inventory

## 5.2.2 Changes of Variable Holding Cost(*h*)

The sensitivity analysis for variable holding cost as follows;

In figure 5.4 shows the sensitivity of variable holding cost, then the effects *TIC* cover the holding cost (*h*) are studied by changing the *h* values over the range from -25% to 100%. If the variable holding cost decrease 20%, it will be decreasing in *TIC* to

12,47% for retroactive holding cost, and 9,21% for incremental holding cost. If the variable holding cost increase 20%, it will be increasing in *TIC* to 11,15% for retroactive holding cost, and 9,21% for incremental holding cost. If the variable holding cost increase 50%, it will be increasing in *TIC* to 21,17% for retroactive holding cost, and 18,41% for incremental holding cost. If the variable holding cost increase 75%, it will be increasing in *TIC* to 30,35% for retroactive holding cost, and 27,62% for incremental holding cost. If the variable holding cost increase 100%, it will be increasing in *TIC* to 38,87% for retroactive holding cost, and 36,82 % for incremental holding cost. It is observed that the total inventory cost are directly related with variable holding cost and its effect becomes decreasing inventory cost if the value of variable holding cost to be decreased and otherwise. Based on the figure 5.5, can be analyzed if the total inventory cost using incremental model still less than using retroactive.

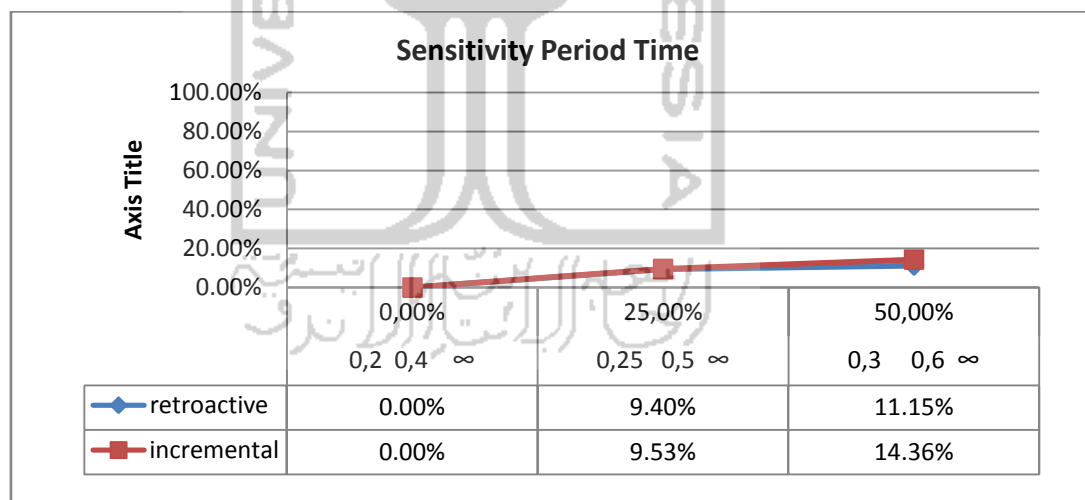


**Figure 5.5** Changes of Variable Holding Cost

### 5.2.2 Changes of Period Time(*t*)

The sensitivity analysis for period time as follows;

In figure 5.4 shows the sensitivity of period time, then the effects *TIC* over the period time ( $t$ ) are studied by changing the  $t$  values over the range from 0% to 50%. If the period time increase 25%, it will be decreasing in *TIC* to 9,40% for retroactive holding cost, and 9,53% for incremental holding cost. If the period time increase 50%, it will be increasing in *TIC* to 11,15% for retroactive holding cost, and 14,36 % for incremental holding cost. It is observed that the total inventory cost are directly related with period time and its effect becomes increasing inventory cost if the value of period time to be increased. Based on the figure 5.5, can be analyzed if the total inventory cost using retroactive model less than using incremental. Because of in the inventory cost of incremental influenced by period time. Thus, if the value of period time increased it also influence the inventory cost of incremental model to increasing.



**Figure 5.6** Changes of period time

## CHAPTER VI

### CONCLUSION AND SUGGESTION

#### 6.1 Conclusion

From all of processes that have been done in this research, based on the problem questions can be concluded that:

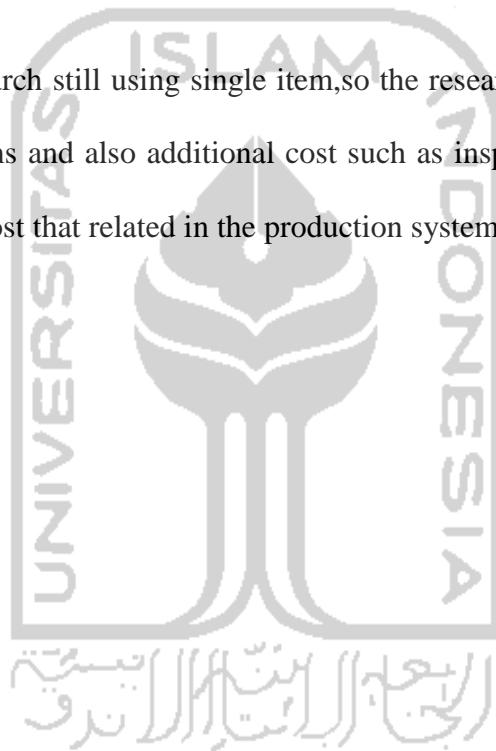
1. The result of retroactive holding cost is optimal order quantity =2068, optimal cycle time =0,35 and total inventory cost= Rp.1.667.583,29. The result of incremental holding cost is optimal order quantity =2369, optimal cycle time=0,4, and total inventory cost =Rp.1.218.979,98/years. Thus, incremental function in holding cost less costly and to be best optimum solution than retroactive function.
2. In planned shortage there can be both time that are time dependent that consider the backorder cost and time independent that consider the administrative backorder cost. The result shows the optimum backorder =360 units, time dependent shortage cost =Rp.1.110.000, and time independent shortage cost =Rp.180.000
3. Based on the sensitivity analysis, the variable holding cost has significant effect to inventory cost. If the variable holding cost increases 50%, it will be increasing in TIC to 21,17% for retroactive holding cost, and 18,41 % for incremental holding cost.



## 6.2 Suggestion

Based on the result of the research and discussion that have been done, can be concluded :

1. In this research, the variable cost using 3 different periods range, those are short term, medium term, and long term. Thus, the research with an additional range of periods and different periods of time will give different results.
2. This research still using single item, so the research need to be continued with multi-items and also additional cost such as inspection cost, rework cost, and another cost that related in the production system



**This thesis, I dedicate to my parents,**

**Thanks to my Dad and Mom for your prayer**

**sacrifice and advices.**



## MOTTO

*“O you, who have believed, seek help through patience and prayer. Indeed, Allah is with the patient.” (Al-Baqarah: 153)*

*“There is no problem that can not be solved as long as there commitment to resolve”*

*“Allah has educated me with the best of education.”*

*(HR. by Ahmad)*



## PREFACE



### **Assalamualikum Wr.Wb**

Alhamdulillah robbil a'lamin. First of all, I would like to say thanks to Allah SWT that always give us bless and mercy for all mankind who had faith and worship to God. Because of His bless and mercy, I can finish my thesis with Title **INVENTORY MODEL BASED WITH VARIABLE HOLDING COST TO ACCOMODATE PLANNED SHORTAGE**. This Thesis is part of requirement to get bachelor's degree of Industrial Engineering Department at University Islam Indonesia.

I would also like to express my profound gratitude for the love, support, and faith to my parents, Nurjono, SH and Puji Rahayu. And I'm so thankful for the happiness brought by my sisters, Galuh Hayu Noormalisa, and my brother, Rizda Noor Ali Wicaksono and Swastika Noor Yudha Pratama

In this opportunity I would like to say special thanks and deepest respect to my supervisor Dr. Mirwan Ushada, S.TP, M.App.Life.Sc that always gives support, advice, guidance, and suggestion to me so that I can finish my thesis. Furthermore, my appreciation goes to CV. Yopan Ceramic for the cooperation as my research object.

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**Wassalamualaikum Wr. Wb**

Yogyakarta, February 30, 2012



Author

## REFERENCE

Alfares, K.H. (2007). Inventory model with stock-level dependent demand rate and variable holding cost. *International journal Production Economics* 108, 259 – 265.

Assauri, Sofjan. (1993). Manajemen Produksi dan Operasi. Edisi Empat. Jakarta : Lembaga Penerbit: Fakultas Ekonomi Universitas Indonesia.

Baiduri. (2002). *Persamaan Diferensial dan Matematika Model*. Malang: UMM press.

Chang, J.H., and Dye, Y.C. (2000). An Inventory Model with Stock-Dependent Demand in Response to a Temporary Sale Price. *Information and Management Sciences* 11, 1-17.

Chapra, S.E., and Canale, R.P. (1990). Numerical Method for Engineers. *McGraw-Hill Inc.* Singapore

Chen, C.K., and Lo, C.C. (2006). Optimal production run length for products sold with warranty in an imperfect production system with allowable shortages. *Mathematical and Computer Modeling* 44, 319–331.

Cheng, et.al. (2007). Optimal replenishment policies with allowable shortages for a product life cycle. *Computers & Mathematics with Applications* 54(10), 1582–1594.

Chung, K.J. (2003). An Algorithm for an inventory model with inventory-level-dependent demand rate. *Computers & Operations Research* (30), 1311–1317.

Hu, F., and Liu, D. (2010). Optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages. *Applied Mathematical Modeling* (34), 3108–3117.

Kelley, C. T., (1995). *Iterative Methods for Linear and Nonlinear Equations*. SIAM, Philadelphia.

Pamuntjak, et.al., (1990). *Persamaan Diferensial Biasa*. Bandung: ITB.

Rangkuti, Fredi.(2000). *Manajemen Persediaan, aplikasi dibidang bisnis*; Jakarta: Radjagrafindo Persada.

Schroeder, R.G. (1994). *Manajemen Operasi : Pengambilan Keputusan dalam Suatu Fungsi Operasi*. Jilid 2. 3<sup>th</sup> Edition. Jakarta : Erlangga.

Shamsi, et.al., (2010). Economic Production Quantity in Reworkable Production Systems with Inspection Errors, Scraps and Backlogging. *Journal of Industrial and Systems Engineering*, 3(3), 170-188.

Shao, Y.E., Fowler, J.W. and Runger, G.C. (2000). Determining the optimal target for a process with multiple markets and variable holding cost. *International Journal of Production Economics*, 65(3), 229-242.

Singh, R.S., Singh, P.A and Bhatia, D. (2010). A Supply Chain Model with Variable Holding Cost for Flexible Manufacturing System. *International Journal of Operations Research and Optimization* 1 (1), 107-120

Sumayang, Lulu. (2003), Dasar-dasar Management Produksi dan Operasi, 1<sup>th</sup> edition. Jakarta: Salemba Empat

Tampubolon, M. P. (2004). Operations Management. Jakarta : Ghalia Indonesia.

Tersine, R. J., (1994). Principles of Inventory and Materials Management. Fourth Edition. Prentice-Hall, Inc. New Jersey.

Xu, Y., and Sarker, R.B. (2003). Models for a family of products with shelf life, and production and shortage costs in emerging markets. *Computers & Operations Research* 30 , 925–938