BAB III

ANALISIS DAN PERHITUNGAN

3.1. Umum

Dalam tugas akhir ini akan dianalisa bentuk plat segi empat dan lingkaran dengan variasi dukungan sesuai dengan batasan sebelumnya.

Pada beberapa kasus khususnya pada kasus-kasus dimana garis luluh ditentukan dengan melalui beberapa dimensi yang tidak diketahui, penyelesaian langsung dengan metode kerja virtuil akan sangat menyulitkan dan harus memakai cara coba-coba. Oleh karena itu dalam penyelesaian kasus ini dipakai metode keseimbangan momen (G. Winter, 1993).

Sedangkan khusus untuk plat berlubang dipakai metode kerja virtuil karena bila memakai metode keseimbangan, momen yang diperoleh tidak dapat mewakili momen pada keseluruhan plat.

Dalam analisa ini ditentukan pola keruntuhan pada plat terlebih dahulu dengan konsep dan rumusan seperti pada pasal terdahulu.

Untuk memudahkan perhitungan dipakai pemisalan dan notasi sebagai berikut:

l_x = bentang plat arah pendek

l_y = bentang plat arah panjang

Ml_x = Momen lentur plat persatuan panjang dilapangan searah bentang l_x

Ml_y = Momen lentur plat persatuan panjang dilapangan searah bentang l_y

Mt_x = Momen lentur plat persatuan panjang ditumpuan searah bentang l_x

Mt_y = Momen lentur plat persatuan panjang ditumpuan searah bentang l_y.

q = beban total terbagi rata.

Pemisalan :

$Ml_x = M$

$Ml_y = \mu\, Ml_x = \mu\, M$

$Mt_x = i\, Ml_x = i\, M$

$Mt_y = i\, Ml_y = i\, \mu\, M$

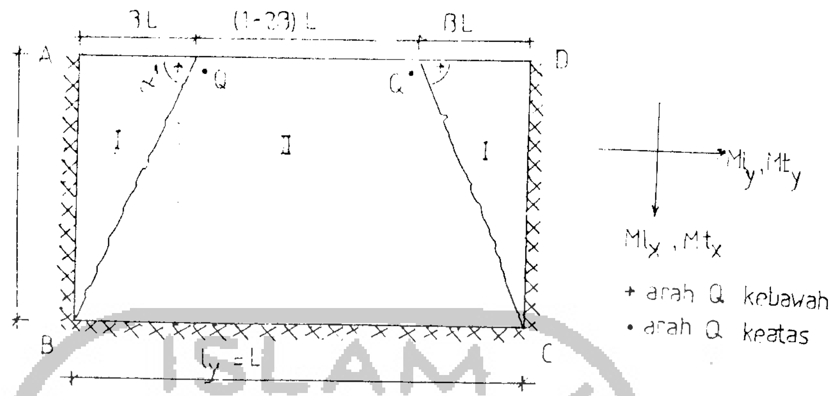
$\frac{l_y}{l_x} = c$

$ly = c\, lx$

Perlu diperhatikan bahwa karena momen luluh merupakan momen utama, sepanjang garis-garis luluh besarnya momen puntir adalah nol dan pada kebanyakan kasus besarnya gaya geser juga sama dengan nol. Untuk itu dalam penulisan persamaan-persamaan keseimbangan yang diperhitungkan hanya momen lentur (M).

3.2. Analisis Dan Perhitungan

3.2.1. Bentuk plat empat persegi panjang dengan tiga isinya terjepit penuh dan satu sisi panjangnya bebas, yang dibebani beban terbagi merata (gambar 3.1.)



Gambar 3.1.

$l_x = \alpha L$

$ly = L$

$$\frac{ly}{l_x} = \frac{L}{\alpha\ L} = c$$

$\alpha = 1/c$

$L = ly = c\ l_x$

Gaya koreksi pada tepi bebas

$$Q = M.ctg\ \alpha' = M\ \frac{\beta L}{\alpha L} = \frac{M\beta}{\alpha}$$

Ditinjau segmen I

Σ momen terhadap sisi AB = 0

$$(Ml_y + Mt_y)\ \alpha\ L = \frac{1}{2}.\alpha L.\beta L.q.\frac{1}{3}\beta L + \frac{M\beta}{\alpha}.\beta L$$

$$(\mu M + i\mu M)\alpha L = \frac{1}{6}\ q\ L^3\alpha\beta^2 + 1/\alpha\ M.L\beta^2$$

$$M\ (\mu\alpha L + iu\alpha L - 1/\alpha\ L\beta^2) = \frac{1}{6}.qL^3\alpha\beta^2$$

$$M\ L/\alpha\ (\mu\alpha^2 + i\upsilon\alpha^2 - \beta^2) = 1/6\ qL^3\alpha\beta^2$$

$$M = \frac{q\ \alpha^2 L^2\beta^2}{6(\mu\alpha^2 + i\mu\alpha^2 - \beta^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Ditinjau segmen II

Σ momen terhadap sisi BC = 0

$$M1_x \;.\; 2\beta L + Mt_x \,.\, L = 2.\frac{1}{2}.\beta L.\alpha L.q.\frac{1}{3}.\alpha L$$

$$+\alpha L(1-2\beta)L.q.\frac{1}{2}\alpha L - 2.\frac{M\beta}{\alpha}\;\alpha L$$

$$2L\beta M + LM = \frac{1}{3}\alpha^2L^3q\beta + \frac{1}{2}\alpha^2L^3q - \alpha^2L^3q\,\beta$$

$$- 2ML\beta$$

$$ML(2\beta + 1 + 2\beta) = \frac{1}{2}\alpha^2L^3q - \frac{2}{3}\alpha^2L^3q\,\beta$$

$$M = \frac{\alpha^2L^3\;q\;(\frac{1}{2} - \frac{2}{3}\beta)}{L\;(1 + 4\beta)}$$

$$M = \frac{\alpha^2L^2\;q\;(\frac{1}{2} - \frac{2}{3}\beta)}{L\;(1 + 4\beta)}$$

$$M = \frac{q\;\alpha^2L^3\;(3 - 4\beta)}{6\;(1 + 4\beta)} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

Persamaan (1) = persamaan (2)

$$\frac{q\alpha^2L^2\beta^2}{6(\mu\alpha^2 + i\mu\alpha^2 - \beta^2)} = \frac{q\alpha^2L^2(3 - 4\beta)}{6\;(1 + 4\beta)}$$

$$\beta^2(1 + 4\beta) = (3 - 4\beta)\;(\mu\alpha^2 + i\mu\alpha^2 - \beta^2)$$

$$4\beta^2 + 4\mu\alpha^2(1 + i)\beta - 3\mu\alpha^2(1 + i) = 0$$

Diambil akar positif

$$\beta = \frac{-4\mu\alpha^2(1 + i) + \sqrt{(4\mu\alpha^2(1 + i))^2 + 4.4.3\mu\alpha^2(1 + i)}}{2.4}$$

$$\beta = -\frac{1}{2}\mu\alpha^2(1 + i) + \frac{1}{8}\sqrt{16\alpha^2(\alpha^2\mu^2(1 + i)^2 + 3\;\mu\;(1 + i)}$$

$$\beta = -\frac{1}{2}\mu\alpha^2(1 + i) + \frac{1}{2}\;\alpha\sqrt{\mu(1 - i)\;(\alpha^2\mu\;(1 + i) + 3)}$$

Jika $A = \sqrt{\mu\;(1 + i)\;(\alpha^2\mu\;(1 + i) + 3)}$

$$A = \sqrt{\mu\ (1 + i)\ (\mu/c^2(1 + i) + 3)}$$

Maka $\beta = \frac{1}{2}\ \alpha\ (A - \mu\alpha(1 + i))$

$$= \frac{1}{2c}(A - \frac{\mu}{c}(1 + i)) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

Persamaan (3) disubstitusikan ke persamaan (2)

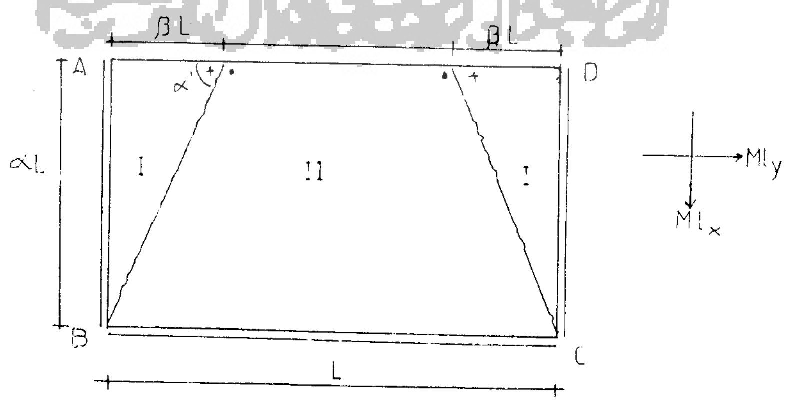
$$M = \frac{q\ \alpha^2 L^2\ (3 - 4\beta)}{6\ (1 + 4\beta)}$$

$$M = \frac{1/c^2\ \{3 - 2/c\ (A - u/c\ (1 + i))\}}{6\{1 + 2/c(A - \mu/c(1 + i))\}} .q l_x^2$$

$$M = \frac{1/c^2\ \{3 - 2/c\ (A - \upsilon/c\ (1 + i))\}}{6/c^2\{c^2 + 2(cA - \mu(1 + i))\}} .q l_x^2$$

$$M = \frac{3\ c^2 - 2\ (cA - \mu(1 + i))}{c^2 + 2(cA - \mu(1 + i))\}} .q l_x^2$$

3.2.2. Plat segi empat dengan tiga sisinya terletak bebas dan satu sisi panjanghya bebas. yang dibebani beban terbagi merata (gambar 3.2).



Gambar 3.2.

Gaya Koreksi $Q = M \text{ ctg } \alpha' = M\ \beta L/\alpha L = M\beta/\alpha$

Ditinjau dari Segmen I

Σ Momen terhadap sisi AB = 0

$$Ml_y . \alpha L = \frac{1}{2}\ \alpha L . \beta L . q . \frac{1}{3}\beta L + M\frac{\beta}{\alpha} . \beta L$$

$$\mu M . \alpha L - \frac{1}{2}\beta L = \frac{1}{6}\ qL^3\alpha\beta^2$$

$$M\ \frac{L}{\alpha}\ (\mu\alpha^2 - \beta^2) = \frac{1}{6}\ L^2\alpha\beta^2$$

$$M = \frac{q\ \alpha^2 L^2\beta^2}{6\ (\mu\alpha^2 - \beta^2)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Ditinjau dari segmen II

Σ momen terhadap sisi BC = 0

$$Ml_x . 2\beta L = 2 . \frac{1}{2}\ \beta L . \alpha L . q . \frac{1}{3}\alpha L + \alpha L(1 - 2\beta)L . q . \frac{1}{2}\alpha L$$
$$-2\ M\beta/\alpha . \alpha L$$

$$M . 2\beta L + 2M\beta L = \frac{1}{3}\alpha^2\ L^3 q\beta + \frac{1}{2}\ \alpha^2 L^3 q - \alpha^3 L^3 q\beta$$

$$4\beta LM = \frac{1}{2}\ \alpha^2 L^3 q - \frac{2}{3}\ \alpha^2 L^3 q\beta$$

$$M = \frac{q\alpha^2 L^2(3 - 4\beta)}{24\beta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Persamaan (1) = persamaan (2)

$$\frac{q\alpha^2 L^2\beta^2}{6(\mu\alpha^2 - \beta^2)} = \frac{2\alpha^2 L^2(3 - 4\beta)}{24\beta}$$

$$4\beta^3 = (\mu\alpha^2 - \beta^2)\ (3 - 4\beta)$$

$$4\beta^3 = 3\mu\alpha^2 - 4\ \mu\alpha^2\beta - 3\beta^2 + 4\beta^3$$

$$3\beta^2 + 4\mu\alpha^2\beta - 3\mu\alpha^2 = 0$$

Diambil akar positif

$$\beta = \frac{-4\mu\alpha \;\mu\alpha^{2} + \sqrt{16\mu^{2}\alpha^{4} + 4.3.3\mu\alpha^{2}}}{2.3}$$

$$\beta = -\frac{2}{3}\mu\alpha^{2} + \frac{1}{3}\sqrt{4\alpha^{2}(4\mu^{2}\alpha^{2} + 9\mu}$$

$$\beta = -\frac{2}{3}\mu\alpha^{2} + \frac{1}{3}\,\alpha\sqrt{\mu(4\mu\alpha^{2}\alpha + 9)}$$

misal : $A = \sqrt{\mu\,(4\mu\alpha^{2} + 9)}$

Diketahui $\alpha = \frac{1}{c}$

$L = c\,lx$

$$A = \sqrt{\mu\,(4\mu/c^{2} + 9)}$$

$$= \frac{1}{c}\sqrt{\mu\,(4\mu + 9c^{2})}$$

maka $\beta = \frac{1}{3}\,c^{2}\,(cA - 2\mu)$ .... (3)

Persamaan (3) disubstisusikan ke persamaan (2)

$$M = \frac{q\;1/c^{2}(clx)^{2}\;(3 - 4\;1/3c^{2}\;(cA - 2\mu))}{24.\frac{1}{3}\,c^{2}\;(CcA - 2\mu)}$$

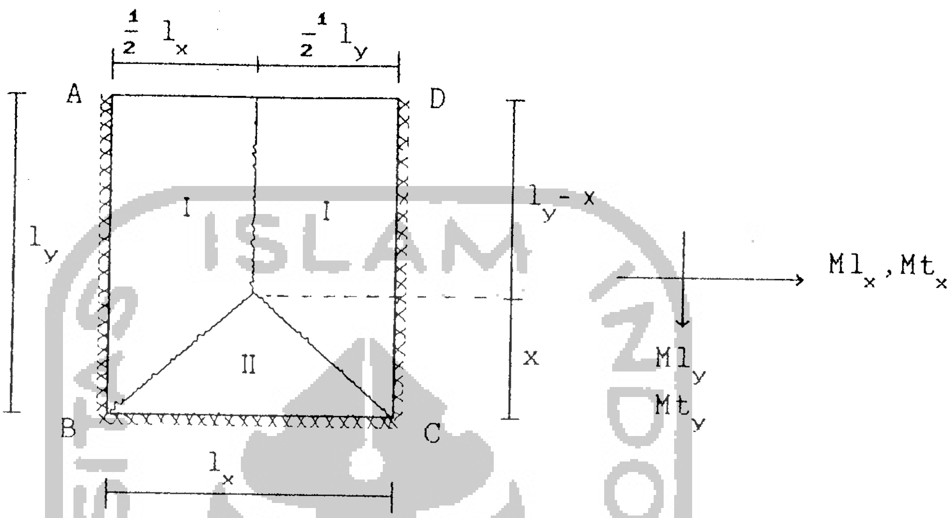
$$M = \frac{\frac{1}{3}\,c^{2}\;(9c^{2} - 4(cA - 2\mu))}{\frac{1}{3}\,c^{2}.24(cA - 2\mu)} = ql_{x}^{2}$$

$$M = \left[\frac{9c^{2} - 4\;(cA - 2\mu)}{24(cA - 2\mu)}\right]\;ql_{x}^{2}$$

Asumsi keluluhan diatas hanya dipakai jika :

$\beta L \leq \frac{1}{2}\,L = \frac{1}{2}\,clx$

3.2.3. Plat segi empat dengan tiga sisinya terjepit penuh dan satu sisi pendeknya bebas, yang dibebani beban terbagi merata (gambar 3.3).



Gambar 3.3

Ditinjau segmen I

Σ Momen terhadap sisi AB = 0

$$(Mt_x + Ml_x)\ l_y = (l_y - \varkappa).\frac{1}{2}\ l_x.q.\frac{1}{4}\ l_x + \frac{1}{2}.x.\frac{1}{2}\ l_x.q.\frac{1}{6}\ l_x$$

$$(i\ Ml_x + Ml_x)\ l_y = \frac{1}{8}\ l_x^2\ l_y\ q - \frac{1}{8}\ l_x^2\ q\varkappa + \frac{1}{24}\ l_x^2 q.\varkappa.$$

$$(i\ M + M)\ l_y = \frac{1}{8}\ l_x^2\ l_y\ q - \frac{1}{12} l_x^2\ q\varkappa$$

$$(i+1)M\ l_y = \frac{1}{8}\ l_x^2\ l_y\ q - \frac{1}{12} l_x^2\ q\varkappa$$

$$(i+1)M = \frac{1}{8}\ l_x^2\ l_y\ q - \frac{1}{12}\left[\frac{l_x}{l_y}\right] l_x.q\varkappa$$

$$(i+1)M = \frac{1}{8}\ l_x^2\ l_y\ q - \frac{1}{12c}\ l_x\ q\varkappa$$

$$M = \frac{1}{(i+1)}\ (\frac{1}{8}\ l_x^2\ q - \frac{1}{12}\ l_x.q\varkappa)\ldots(1)$$

Di tinjau segmen II

Σ Momen terhadap sisi BC = 0

$$(Mt_y + Ml_x) \quad l_y = \frac{1}{2}\, l_x . \varkappa . q . \frac{1}{3}\varkappa$$

$$(i\ Ml_x + Ml_x)\ l_y = \frac{1}{6}\, q\, \varkappa^2\, l_x$$

$$(i\ \mu M + \mu M) = \frac{1}{6}\, q\, \varkappa^2$$

$$M(i\mu + \mu) = \frac{1}{6}\, q\, \varkappa^2$$

$$M = \frac{1}{\mu(i+1)}\left(\frac{1}{6}\, q\, \varkappa^2\right) \ldots \quad (2)$$

Persamaan (1) = persamaan (2)

$$\frac{1}{(i+1)}\left(\frac{1}{8}\, l_x^{\,2} q - \frac{1}{12c}\, l_x\, q\, \varkappa\right) = \frac{1}{\mu(i+1)} \cdot \frac{1}{6}\, q\, \varkappa^2$$

$$\left(\frac{1}{6\mu}\, q\right)\varkappa^2 + \left(\frac{1}{12c}\, l_x\, q\right)\varkappa - \frac{1}{8}\, l_x^{\,2} q = 0$$

$$\left(\frac{1}{6\mu}\right)\varkappa^2 + \left(\frac{1}{12c}\, l_x\right)\varkappa - \left(\frac{1}{8}\, \varkappa^2\right) = 0$$

diambil akar yang positif

$$\varkappa = \frac{\left(-\frac{1}{12c}\, l_x\right) + \sqrt{\left(\frac{1}{12c}\, l_x\right)^2 - 4\left(\frac{1}{6\mu}\right)\left(-\frac{1}{8}\, l_x^{\,2}\right)}}{2 . \frac{1}{6\mu}}$$

$$\varkappa = -\frac{\mu}{4c}\, l_x + 3\mu \sqrt{\left(\frac{1}{144c^2} + \frac{1}{12\mu}\right) l_x^{\,2}}$$

$$\varkappa = -\frac{\mu}{4c}\, l_x + 3\mu\, l_x \sqrt{\left(\frac{12c + \mu}{144\mu c^2}\right)}$$

$$\varkappa = -\frac{\mu}{4c}\, l_x + \frac{3\mu\, l_x}{12c} \sqrt{\left(\frac{12c + \mu}{\mu}\right)}$$

$$\varkappa = -\frac{\mu}{4c}\, l_x + \frac{\mu}{4c}\, l_x \sqrt{\left(\frac{12c}{\mu} + 1\right)}$$

$$\varkappa = \frac{\mu}{4c}\, l_x \left[\sqrt{\left(\frac{12c}{\mu} + 1\right)} - 1\right]$$

$$\text{misal } A = \sqrt{\left(\frac{12c}{\mu} + 1\right)} - 1$$

$$\text{maka } \varkappa = \frac{\mu}{4c}\, l_x \cdot A \ldots\ldots\ldots\ldots \quad (3)$$

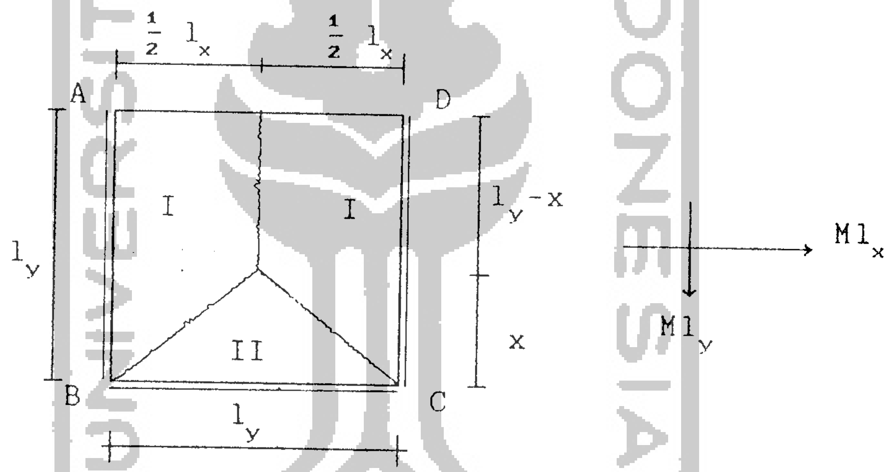
Persamaan (3) disubstitusikan ke persamaan (2)

$$M = \frac{1}{6\mu(i+1)} q \left(\frac{\mu}{4c} l_x . A\right)^2$$

$$M = \frac{1}{6\mu(i+1)} q \frac{\mu^2}{16c^2} l_x^2 . A^2$$

$$M = \frac{\mu A^2}{96c^2(i+1)} q\, l_x^2$$

3.2.4. Plat segi empat dengan tiga sisinya terletak bebas dan satu sisi pendeknya bebas, yang dibebani beban terbagi merata (gambar 3.4).



Gambar 3.4

Ditinjau segmen I

Σ Momen terhadap sisi AB = 0

$$Ml_x . Ml_y = (l_y - x).\frac{1}{2} l_x .q \frac{1}{4} l_x + \frac{1}{2} x \frac{1}{2} l_x\ q.\frac{1}{6} l_x$$

$$M.\ l_y = \frac{1}{8} l_y\ l_x^2 q - \frac{1}{8} l_x^2 qx + \frac{1}{24} l_x^2 qx$$

$$M.\ l_y = \frac{1}{8} l_y\ l_x^2 q - \frac{1}{12} l_x^2 qx$$

$$M = \frac{1}{8} l_y\ l_x^2 q - \frac{1}{12}\left[\frac{l_x}{l_y}\right] l_y\ qx$$

$$= \frac{1}{8} l_y \; l_x^{\,2} q - \frac{1}{12c} . l_x \; q . x \quad ....(1)$$

Ditinjau segmen I

Σ Momen terhadap sisi BC = 0

$$M l_y . l_x = \frac{1}{2} \; l_x . x . q . \frac{1}{3} \; x$$

$$\mu M = \frac{1}{6} \; q \; x^2$$

$$M = \frac{1}{6\mu} \; q^2 x \quad ....(2)$$

Persamaan (1) = persamaan (2)

$$\frac{1}{8} \; l_x^{\,2} q - \frac{1}{12c} \; l_x \; q \; x = \frac{1}{6\mu} \; q \; x^2$$

$$\frac{1}{6\mu} \; q \; x^2 + \frac{1}{12c} \; l_x \; q \; x - \frac{1}{8} \; l_x^{\,2} \; q = 0$$

$$\frac{1}{6\mu} \; x^2 + \frac{1}{12c} \; l_x \; x - \frac{1}{8} \; l_x^{\,2} = 0$$

$$x^2 + \frac{\mu}{2c} \; l_x \; x - \frac{3\mu}{4} \; l_x^{\,2} = 0$$

Diambil akar positif

$$x = \frac{-\frac{\mu}{2c} \; l_x + \sqrt{\left(\frac{x}{2c} l_x\right)^2 + 4 . \frac{3\mu}{4} . l_x^{\,2}}}{2.1}$$

$$x = -\frac{\mu}{4c} \; l_x + \frac{1}{2} \; l_x \sqrt{\frac{\mu^2}{4c^2} + 3\mu}$$

$$x = \frac{1}{2} \; l_x \left[\sqrt{\frac{\mu^2}{4c^2} + 3\mu} - \frac{\mu}{2c} \right]$$

Misal $A = \left[\sqrt{\frac{\mu^2}{4c^2} + 3\mu} - \frac{\mu}{2c} \right]$

Jadi $x = \frac{1}{2} \; l_x \; A$ .... (3)

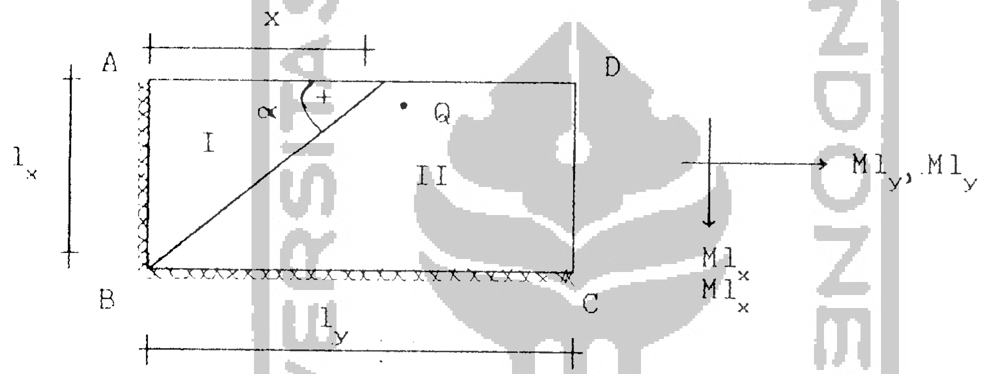
Persamaan (3) disubstitusikan ke persamaan (2)

$$M = \frac{1}{6}\, q \,(\frac{1}{2}\, l_x\, A)^2$$

$$M = \frac{1}{24}\, A^2\, q\, l_x$$

3.2.5. Plat segi empat yang kedua sisinya terjepit penutuppada sisi pendek dan panjangnya dan sisi yang lain bebas, yang dibebani beban terbagi merata.

a. Asumsi keluluhan pertama (gambar 3.5).



Gambar 3.5

Gaya Koreksi : $Q = M \text{ ctg } \alpha$

$$Q = M \frac{x}{l_x}$$

Ditinjau segmen I

Σ Momen terhadap sisi AB = 0

$$(Ml_x + Ml_y)\, l_x = \frac{1}{2}\, l_x \,.\, x \,.\, q \frac{1}{3}\, x + (M \frac{x}{l_x})\, x$$

$$(\mu M + i\mu M)\, l_x - M \frac{x^2}{l_x} = \frac{1}{6}\, l_x\, q\, x^2$$

$$M \left[\frac{1}{l_x} (\mu l_x^2 + i\mu l_x^2 - x^2) \right] = \frac{1}{6}\, l_x\, q\, x^2$$

$$M = \frac{q \,.\, l_x^2 \,.\, x^2}{6(\mu l_x^2 + i\mu l_x^2 - x^2)} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Ditinjau dari segmen II

Σ Momen terhadap sisi BC = 0

$$Ml_x . \varkappa + Mt_x . l_y = \frac{1}{2} \; l_x . \varkappa . q . \frac{1}{3} \; l_x + (l_x - \varkappa) l_x . \frac{1}{2} \; l_x . q - (M \; \frac{x}{l_x}) l_x$$

$$M . \varkappa + iM . c . l_x = \frac{1}{6} \; l_x^{\,2} qx + \frac{1}{2} \; l_y \; l_x^{\,2} q - \frac{1}{2} \; l_x^{\,2} qx - M_x$$

$$M(2\varkappa + i . c . l_x) = \frac{1}{2} \; c . l_x^{\,3} q - \frac{1}{2} \; l_x^{\,2} . q . \varkappa$$

$$M = \frac{q . l_x^{\,2} (\frac{1}{2} \; cl_x - \frac{1}{3} \; \varkappa)}{(2x + i . c . l_x)} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

Persamaan (1) = persamaan (2)

$$\frac{q \; l_x^{\,2} . \varkappa^2}{6(\mu l_x^{\,2} + i\mu l_x^{\,2} - x^2)} = \frac{q \; l_x^{\,2} (\frac{1}{2} \; cl_x - \frac{1}{3} \; \varkappa)}{(2x + ic . l_x)}$$

$$6(\mu l_x^{\,2} + i\mu l_x^{\,2} - x)(\frac{1}{2} \; cl_x - \frac{1}{3} \; \varkappa) = \varkappa^2 (2x + ic . l_x)$$

$$3\mu l_x^{\,3} - 2\mu l_x^{\,2} \varkappa + 3i . \mu . cl_x^{\,3} - 2i\mu l_x^{\,2} \varkappa - 3cl_x . x^2 + 2x^3$$

$$- 2\varkappa^3 + icl_x . x^2 = 0$$

$$-(3cl_x + icl_x) \varkappa^2 - (2\mu l_x^{\,2} + 2 . i\mu l_x^{\,2}) x$$

$$+ (3\mu cl_x^{\,3}) = 0$$

Diambil akar yang positif :

$$x = \frac{2l_x^{\,2} \mu(1+i) + \sqrt{2l_x^{\,2} \mu(1+i)^2 + \left[(3cl_x + icl_x)(3\mu cl_x^{\,3})\right]}}{-2 \; (3cl_x + icl_x)}$$

$$x = \frac{2l_x^{\,2} \mu(1+i)}{2l_x \; c(3+1)} + \frac{\sqrt{4l_x^{\,4} \mu^2 (1+i) + 36c^2 \mu l_x^{\,4} (3+i)}}{2cl_x (3+i)}$$

$$x = \frac{l_x^{\,2} \mu(1+i)}{l_x \; c(3+1)} + \frac{2l_x^{\,2} \sqrt{\mu^2 (1+i)^2 + 9c^2 \mu(3+i)}}{2cl_x (3+i)}$$

misal : $A = \sqrt{\mu^2(1+i)+9c^2\mu(3+1)}$

Jadi :

$$x = \frac{l_x}{c(3+i)}\,(A - \mu(1+i))$$

$$x = \frac{l_x\;B}{c(3+i)}$$

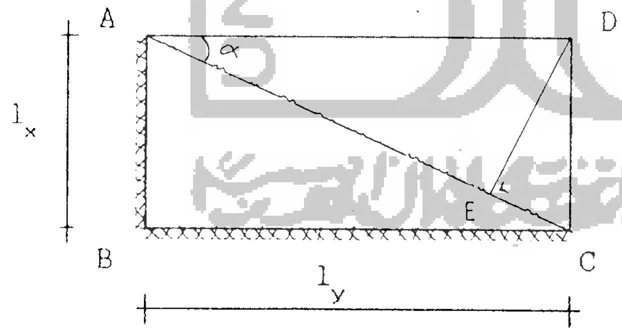
$$x = B = A - \mu(1+i) \;\ldots\ldots\ldots\ldots\; (3)$$

Persamaan (3) disubtitusikan ke persamaan (2)

$$M = \frac{q.l_x^{\,2}\left[\frac{1}{2}c\,l_x - \frac{B\,l_x}{c(3+1)}\right]}{\frac{2.B\;l_x}{c(3+1)} + i.c.l_x}$$

$$M = \left[\frac{\frac{1}{2}.c^2(3+i) - \frac{1}{3}B}{2.B + i.c}\right] q\;l_x^{\,2}$$

b. Asumsi keluluhan kedua (gambar 3.6).



Gambar 3.6

$$\overline{AC} = l_x \,/\, \sin\alpha$$

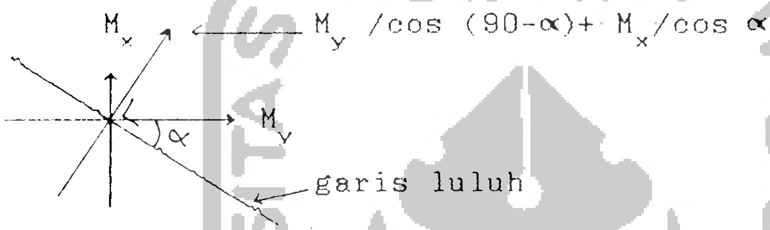
$$\sin\alpha = \frac{l_x}{\overline{AC}} = \frac{l_x}{\sqrt{l_x^{\,2}+l_y^{\,2}}} = \frac{l_x}{\sqrt{l_x^{\,2}+c^2 l_y^{\,2}}}$$

$$= \frac{l_x}{l_x\sqrt{1+c^2}}$$

$$= \frac{1}{\sqrt{1+c^2}}$$

$$\cos\alpha = \frac{c\,.l_x}{l_x\sqrt{1+c^2}} = \frac{c}{\sqrt{1+c^2}}$$

Σ Momen terhadap AC = 0



$$\left[\frac{Ml_y + Mt_y}{\cos(90-\alpha)} + \frac{Ml_x + Mt_x}{\cos\,\alpha}\right]\overline{AC} = \frac{1}{2}\,l_x\,.\,l_y\,.q.\frac{1}{3}\,\overline{DE}$$

$$\left[\frac{\mu M + i\mu M}{\sin\,\alpha} + \frac{M + iM}{\cos\,\alpha}\right]\frac{l_x}{\sin\alpha} = \frac{1}{6}\,l_x\,.cl_x\,.q\,.cl_x\,\sin\,\alpha$$

$$\left[\frac{\mu(1+1)\cos\alpha + (1+i)\sin\alpha}{\sin\,\alpha.\cos\,\alpha}\right]M = \frac{1}{6}\,c^2\,\sin^2\alpha\;q\;l_x^{\,2}$$

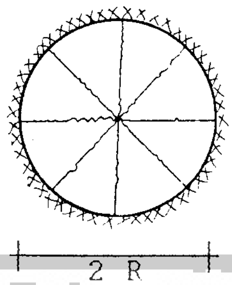
$$M = \frac{c^2\sin^2\alpha\;q\;l_x^{\,2}\;\sin\,\alpha\;\cos\,\alpha}{6(1+i)(\mu co\alpha + \sin\,\alpha)}$$

$$M = \frac{c^2\left(\frac{1}{\sqrt{1+c^2}}\right)^2\left(\frac{1}{\sqrt{1+c^2}}\right)\left(\frac{1}{\sqrt{1+c^2}}\right)}{6(1+i)\left[\mu\frac{c}{\sqrt{1+c^2}} + \frac{1}{\sqrt{1+c^2}}\right]}\,ql_x^{\,2}$$

$$M = \frac{c^2\left(\frac{1}{1+c^2}\right)\left(\frac{c}{1+c^2}\right)}{6(1+i)\left[\frac{\mu c\sqrt{1+c^2} + \sqrt{1+c^2}}{(1\;+\;c^2)}\right]}\,ql_x^{\,2}$$

$$M = \frac{c^2\;q\;l_x^{\,2}}{6(1+i)(1+c^2)(\mu c+1)\sqrt{1+c^2}}$$

3.2.6. Plat lingkaran yang semua sisinya terjepit penuh, yang dibebani beban terbagi rata (gambar 3.7).

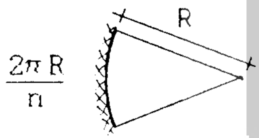


$Ml_x = Ml_y$

Gambar 3.7

Plat terbagi menjadi n segmen

diambil satu segmen



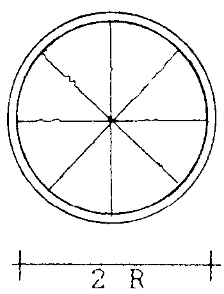
Σ Momen terhadap sisi AB = 0

$$(Mt + Ml) . \frac{2\pi R}{n} = \frac{\pi R^2}{n} . q . \frac{1}{3} R$$

$$M(1 + i) = \frac{1}{6} q R^2$$

$$M = \frac{1}{6(1+i)} . q . R^2$$

3.2.7. Plat lingkaran yang semua sisinya terletak bebas, yang dibebani beban terbagi merata.

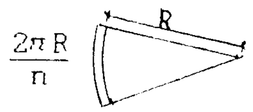


$Ml_x = Ml_y$

Gambar 3.8

Plat terbagi menjadi n segmen

Diambil satu segmen :



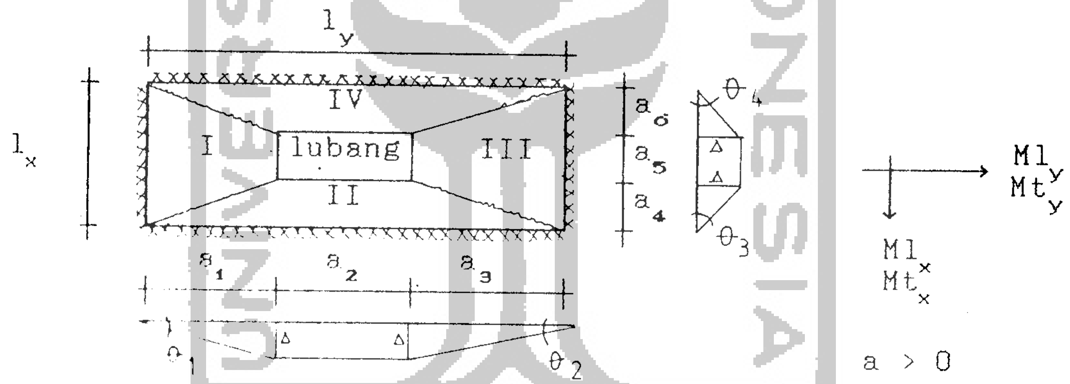
$Ml_x = Ml_y$

Σ Momen terhadap sisi AB = 0

$$Ml \ \frac{2\pi R}{n} = \frac{\pi R^2}{n} \ .q.\frac{1}{3}.R$$

$$M = \frac{1}{6} \ .q.R^2$$

3.2.8. Plat segi empat berlubang yang semua tepinya terjepit (gambar 3.9).



Gambar 3.9

Lendutan virtuil : Δ = 1

- Rotasi masing-masing segmen plat

$$\theta_1 = \frac{\Delta}{a_1} = \frac{1}{a_1} \quad ; \quad \theta_3 = \frac{\Delta}{a_4} = \frac{1}{a_4}$$

$$\theta_2 = \frac{\Delta}{a_3} = \frac{1}{a_3} \quad ; \quad \theta_4 = \frac{\Delta}{a_6} = \frac{1}{a_6}$$

1. Kerja dalam total = Σ Wi

I. $Wi_1 = (Ml_y(a_4+a_6)+ Mt_y . l_x)\theta_1$

$$= M(\mu \ (a_4+a_6)+ \mu i \ l_x) \ \frac{1}{a_1}$$

II. $Wi_2 = (Ml_x(a_1+a_3)+ Mt_x . l_y)\theta_3$

$$= M((a_1+a_3)+ \; i \; l_y) \frac{1}{a_4}$$

III. $Wi_3 = (Ml_y(a_4+a_6)+ Mt_y . l_x)\theta_2$

$$= M(\mu(a_4+a_6)+ \mu i \; l_x) \frac{1}{a_3}$$

IV. $Wi_4 = (Ml_x(a_1+a_3)+ Mt_x . l_y)\theta_4$

$$= M((a_1+a_3)+ \; i \; l_y) \frac{1}{a_6}$$

$\Sigma Wi = Wi_1+ Wi_2+ Wi_3+ Wi_4$

$$= M\left[(a_1+a_3)+il_y)(\frac{1}{a_4} + \frac{1}{a_6})+ \mu(a_4+a_6)+ il)(\frac{1}{a_3} + \frac{1}{a_6})\right]$$

2. Kerja luar total = ΣWe

I. $We_1 = a_1 a_5 q \frac{1}{2} \Delta + \frac{1}{2} a_1 a_4 q . \frac{1}{3} \Delta + \frac{1}{2} a_1 a_6 q \frac{1}{3} \Delta$

$$= \frac{1}{6} q(3a_1 a_5 + a_1 a_4 + a_1 a_6)$$

II. $We_2 = a_2 a_4 q \frac{1}{2} \Delta + \frac{1}{2} a_1 a_4 q . \frac{1}{3} \Delta + \frac{1}{2} a_3 a_4 q \frac{1}{3} \Delta$

$$= \frac{1}{6} q(3a_2 a_4 + a_1 a_4 + a_3 a_4)$$

III. $We_3 = a_3 a_5 q \frac{1}{2} \Delta + \frac{1}{2} a_3 a_4 q . \frac{1}{3} \Delta + \frac{1}{2} a_3 a_6 q \frac{1}{3} \Delta$

$$= \frac{1}{6} q(3a_3 a_5 + a_3 a_4 + a_3 a_6)$$

IV. $We_4 = a_2 a_6 q \frac{1}{2} \Delta + \frac{1}{2} a_1 a_6 q . \frac{1}{3} \Delta + \frac{1}{2} a_3 a_6 q \frac{1}{3} \Delta$

$$= \frac{1}{6} q(3a_2 a_6 + a_1 a_6 + a_3 a_6)$$

$\Sigma We = We_1 + We_2 + We_3 + We_4$

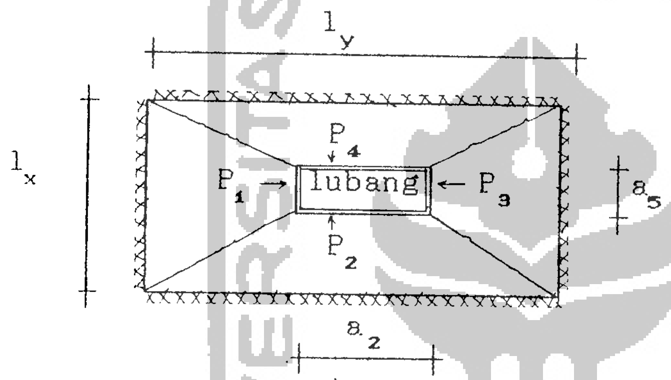
$$= \frac{1}{6} q(3a_1 a_5 + 2a_1 a_4 + 2a_1 a_6 + 3a_2 a_4 + 2a_3 a_4 + 3a_3 a_5 + 2a_3 a_6)$$

Prinsip kerja virtuil

Σ Wi = Σ We

$$M = \frac{1}{6}\, q \, . \frac{3(a_1 a_5 + a_2 a_4 + a_3 a_5) + 2(a_1 a_4 + a_1 a_6 + a_3 a_4 + a_3 a_6)}{((a_1 + a_3) + il_y)(\frac{1}{a_4} + \frac{1}{a_6}) + \mu(a_4 + a_6) + il_x)(\frac{1}{a_1} + \frac{1}{a_6})}$$

Bila dipinggir lobang terdapat beban garis (gambar 3.9a), maka :



Gambar 3.9a.

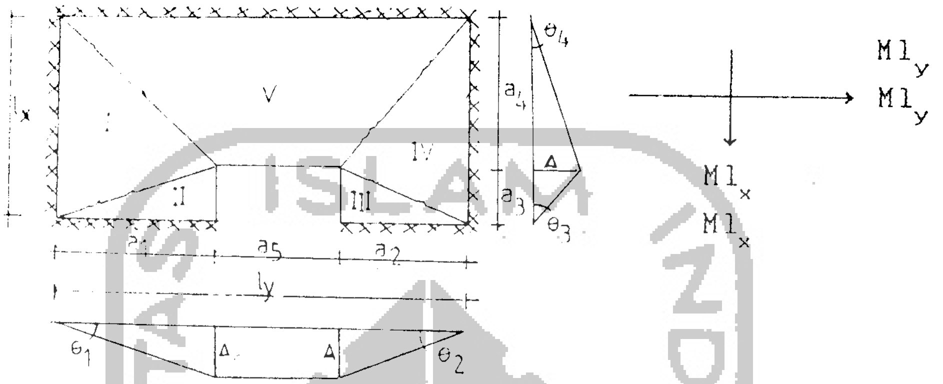
$$P \text{ total} = (P_1 + P_3)a_5\Delta + (P_2 + P_4)a_2\Delta$$

$$= (P_1 + P_3)a_5 + (P_2 + P_4)a_2$$

$$M = \frac{\frac{1}{6}\, q \left[3(a_1 a_5 + a_2 a_4 + a_3 a_5) + 2(a_1 a_4 + a_1 a_6 + a_3 a_4 + a_3 a_6)\right]}{(a_1 + a_3) + il_y)(\frac{1}{a_4} + \frac{1}{a_6}) + \mu(a_4 + a_6) + il_x)(\frac{1}{a_1} + \frac{1}{a_6})}$$

$$+ \frac{(P_1 + P_3)a_5 + (P_2 + P_4)a_2}{(a_1 + a_3) + il_y)(\frac{1}{a_4} + \frac{1}{a_6}) + \mu(a_4 + a_6) + il_x)(\frac{1}{a_1} + \frac{1}{a_6})}$$

3.2.9. Plat segiempat dengan lubang di tepi yang semua sisinya terjepit penuh dan dibebani beban terbagi merata (gambar 3.10.)



Gambar 3.10.

Lendutan virtual = 1

Rotasi pada masing-masing segmen plat :

$$Ø_1 = \frac{\Delta}{a_1} = \frac{1}{a_1}$$

$$Ø_2 = \frac{\Delta}{a_2} = \frac{1}{a_2}$$

$$Ø_3 = \frac{\Delta}{a_3} = \frac{1}{a_3}$$

$$Ø_4 = \frac{\Delta}{a_4} = \frac{1}{a_4}$$

1. Energi dalam = ΣWi

I. $Wi_1 = (Ml_y + Mt_y)\ l_x . Ø_1$

$$= M(\mu+\mu i) l_x \frac{1}{a_1}$$

II. $Wi_2 = (Ml_x + Mt_x)\ a_1 . Ø_3$

$$M(\mu+\mu i) a_1 \frac{1}{a_3}$$

III. $Wi_3 = (Ml_x + Mt_x)\ a_2 . \emptyset_3$

$= M(\mu+\mu i)a_2 \ \frac{1}{a_3}$

IV. $Wi_4 = \left[Ml_y + Mt_y\right]\ l_x . \emptyset_2$

$= M(\mu+\mu i)l_x \ \frac{1}{a_2}$

V. $Wi_5 = \left[Ml_x\left[a_1+a_2\right] + Mt_x\ l_y\right]\ \emptyset_4$

$= M((a_1+a_2)+\ i\ ly)\ \frac{1}{a_4}$

$$\Sigma Wi = Wi_1+Wi_2+Wi_3+Wi_4+Wi_5$$

$$= M\mu(1+i)\frac{lx}{a_1} + M\mu(1+i)\frac{1}{a_3} + M\mu(1+i)\frac{a_2}{a_3}$$

$$+ M\mu(1+i)\frac{lx}{a_2} + M+\frac{(a_1+a_2)}{a_4} + M\frac{i\ ly}{a_4}$$

$$= M\left\{\mu(+i)\frac{lx}{a_1} + \mu\ (1+i)\frac{lx}{a_2} + \frac{i\ ly}{a_4} + (1+i)\ \frac{{}_1a}{a_3}\right.$$

$$\left. +(1+i)\frac{a_2}{a_3} + \frac{(a_1+a_2)}{a_4}\right\}$$

$$= M\left\{\mu(1+i)\left[\frac{1}{a_1} + \frac{1}{a_2}\right] l_x + \frac{i\ ly}{a_4} + \frac{1}{a_3}\ (1+i)\right.$$

$$\left.(a_1+a_2) + \left[\frac{a_1+a_2}{a_4}\right]\right\}$$

$$= M\left\{\left[\mu(1+i)\left[\frac{1}{a_1} + \frac{1}{a_2}\right]\ l_x + \frac{i\ c}{a_4}\right] + l_x\right.$$

$$\left. + \left[\frac{1}{a_3}\ (1+i) + \frac{1}{a_4}\right]\ \left[a_1+a_2\right]\right\}$$

2. Energi Luar : ΣWe

I. $We_1 = \frac{1}{2} . a_1 . l_x . q . \frac{1}{3}\ \Delta$

$= \frac{1}{6}\ q\ a_1\ l_x$

II. $We_2 = \frac{1}{2} . a_1 . a_3 . q . \frac{1}{3} \Delta$

$= \frac{1}{6} . q . a_1 . a_3$

III. $We_3 = \frac{1}{2} . a_2 . a_3 . q . \frac{1}{3} \Delta$

$= \frac{1}{6} . q . a_2 . a_3$

IV. $We_4 = \frac{1}{2} . a_1 . l_x . q . \frac{1}{3} \Delta$

$\frac{1}{6} . q . a_2 . l_x$

V. $We_5 = a_4 . a_5 . q . \frac{1}{2} \Delta + \frac{1}{2} \Delta . a_1 . a_4 . q . \frac{1}{3} \Delta + \frac{1}{2} \Delta . a_2 . a_4 . q . \frac{1}{3} \Delta$

$= \frac{1}{2} . q . a_4 . a_5 . \frac{1}{6} . q(a_2 + a_2) a_4$

$\Sigma We = Wi_1 + Wi_2 + Wi_3 + Wi_4 + Wi_5$

$\Sigma We = \frac{1}{6} q \left(a_1 . l_x + a_1 . a_3 + a_2 . l_x + (a_1 . a_2) . a_4\right) + \frac{1}{2} q . a_4 . a_5$

$= \frac{1}{6} q \left\{(a_1 + a_2) l_x + a_1 + a_3 + (a_1 + a_2) a_4 + 3a_4 \ a_5\right\}$

$= \frac{1}{6} q \left\{(l_x + a_4)(a_1 + a_2) + a_1 + a_3 + 3a_4 \ a_5\right\}$

Syarat keseimbangan energi $\Sigma Wi = \Sigma We$

$M = \frac{1}{6} q \left\{\left(\mu(1+i)\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \frac{ic}{a_4}\right) l_x + \left(\frac{1}{a_3}(1-i) + \frac{1}{a_4}\right)(a_1 + a_2)\right\}$

$= \frac{1}{6} q \left\{(l_x + a_4)(a_1 + a_2) + a_1 a_3 + 3a_4 a_5\right\}$

$$M = \frac{1}{6} q \frac{(l_x + a_4)(a_1 + a_2) + a_1 a_3 + 3a_4 a_5}{\mu(1+i)\left(\frac{1}{a_1} + \frac{1}{a_2}\right) l_x + \frac{i \ l_y}{a_4} + \left[\frac{1}{a_3}(1-i) + \frac{1}{a_4}\right](a_1 + a_2)}$$

Diketahui bahwa $l_x = (a_3 + a_4)$, maka

$$M = \frac{1}{6} q \frac{(a_1 + a_2)(a_3 + 2a_4) + (a_1 a_3) + (3a_4 a_5)}{\mu(1+i)\left(\frac{1}{a_1} + \frac{1}{a_2}\right)(a_3 a_4) + \frac{1}{a_4}(a_1 + a_2 + a_5) + \left[\frac{1}{a_3}(1-i) + \frac{1}{a_4}\right](a_1 + a_2)}$$

Diketahui bahwa $l_x = (a_3 + a_4)$, maka :

$$M = \frac{1}{6}\, q \,\frac{(a_1+a_2)\ (a_3+2a_4)+(a_1 a_3)+(3a_4 a_5)}{\mu(1+i)(\frac{1}{a_1}+\frac{1}{a_2})(a_3 a_4)+\frac{i}{a_4}(a_1+a_2+a_5)+\left[\frac{1}{a_3}(1+i)+\frac{1}{a_4}\right](a_1+a_2)}$$

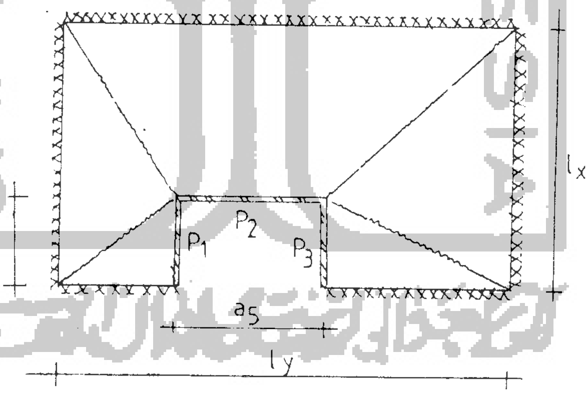
misal :

$$A = (a_1+a_2)(a_3+2a_4)+(a_1 a_3)+(3a_4 a_5)$$

$$B = \left[\mu(1+i)(\frac{1}{a_1}+\frac{1}{a_2})(a_3+a_4)+\frac{i}{a_4}(a_1+a_2+a_5)+\left[\frac{1}{a_3}(1+i)+\frac{1}{a_4}\right](a_1+a_2)\right]$$

Maka : $M = \frac{1}{6}\, q\, \frac{A}{B}$

Bila dipinggir lubang terdapat beban garis (gambar 3.10a), maka :



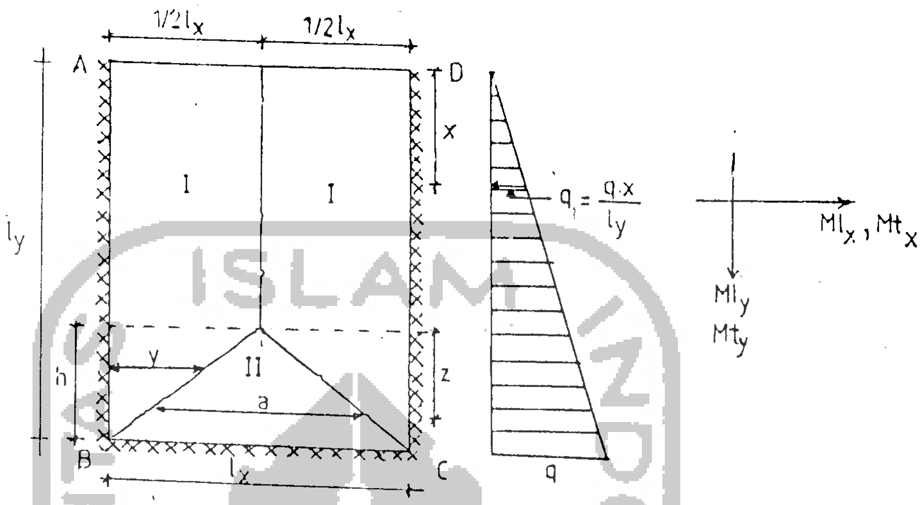
Gambar 3.10a

$$P\ total = (P_1+P_3).a_3.\Delta+P_2.a_5.\Delta$$

$$= (P_1+P_3).a_3+P_2.a_5$$

$$M = \frac{1}{6}.q.\frac{A}{B} + \frac{P\ total}{B}$$

3.2.10. Plast segi empat yang ketiga sisinya terjepit sempurna satu sisiny,yang dibebani beban segi tiga (gambar 3.11).



Gambar 3.11.

(i) $y = \frac{1}{2}\left(\frac{l_y - x}{h}\right) l_x$

(ii) $z = (h - ly) + x$

$a = z \cdot \frac{l_x}{h} = (h - l_y)\ \frac{l_x}{h}$

Ditinjau segmen plat I

Σ momen terhadap sisi AB = 0

$$(Ml_x + Mt_x) \cdot l_y = \int_0^{(l_y - h)} q_1 \cdot \frac{l_x}{2} \cdot \frac{1}{2} \cdot \frac{l_x}{2}\, dx + \int_{(l_y - h)}^{y} q_1 \cdot y \cdot \frac{1}{2}\, y\, dx$$

$$M(1+i) l_y = \frac{1}{8} \int_0^{(l_y - h)} \frac{q_x}{l_x} l_x^2\, dx + \frac{1}{8} \int_{(l_y - h)}^{l_y} \frac{q - x}{l_y} \left(\frac{l_y - x}{h} \cdot l_x\right)^2 dx$$

$$= \frac{q l_x^2}{8 l_y} \int_0^{(l_y-h)} x \, dx + \frac{q l_x^2}{8 l_y . h^2} \int_{(l_y-h)}^{l_y} x(l_y - x) dx$$

$$= \frac{q l_x^2}{8 l_y} \left[\left[\frac{1}{2} x^2 \right]_0^{(l_y-h)} + \frac{1}{h^2} \left(\frac{1}{2} x^2 - \frac{1}{3} x^3 \right) \Big]_{(l_y-h)}^{l_y} \right]$$

$$= \frac{q l_x^2}{8 l_y} \left\{ \frac{1}{2} (l_y - h)^2 + \frac{1}{h^2} \left[\frac{1}{2} l_y \left[l_y^2 - (l_y - h)^2 \right] - \frac{1}{3} \left[l_y^3 - (l_y - h)^3 \right] \right] \right\}$$

$$= \frac{q l_x^2}{8 l_y} \left\{ \frac{1}{2} l_y - l_y h^2 + \frac{1}{2} h^2 + \frac{1}{h^2} \left\{ \frac{1}{2} l_y \left[l_y^2 - l_y^2 + l_y . h - h^2 \right] \right. \right.$$

$$\left. \left. - \frac{1}{3} \left[l_y^3 - l_y h^3 + l_y^2 h - 3 l_y h^2 + h^3 \right] \right\} \right\}$$

$$= \frac{q l_x^2}{8 l_y} \left[\frac{1}{2} l_y^2 - l_y h + \frac{1}{2} h^2 - \frac{3}{2} l_y + \frac{1}{3} h \right]$$

$$M = \frac{1}{(1+i)} \frac{q l_x^2}{8 . l_y^2} \left[\frac{1}{2} h^2 + \left(\frac{1}{3} - l_y \right) h + \left(\frac{1}{2} l_y^2 - \frac{3}{2} l_y \right) \right]$$

$$l_y = c l_x$$

$$M = \frac{q}{8 c^2 (1+i)} \left[\frac{1}{2} h^2 + \left(\frac{1}{3} - c l_x \right) h + \left(\frac{1}{2} c^2 l_x^2 - \frac{3}{2} l_y \right) \right] \quad \ldots\ldots\ldots \; (1)$$

Ditinjau segemen plat II

Σ momen terhadap sisi BC = 0

$$(Ml_y + Mt_y)l_x = \int_{(l_y-h)}^{l_y} q_1 . a . (l_y - x)dx$$

$$M(\mu(1+i))l_x = \int_{(l_y-h)}^{l_y} \frac{q . x}{l_y} . (h - l_y + x) \frac{l_x}{h} (a - x)dx$$

$$= \frac{q}{h . l_y} \int_{(l_y-h)}^{l_y} x(h - l_y + x)(l_y - x)dx$$

$$= \frac{q}{h . l_y} \int_{(l_y-h)}^{l_y} (h . l_y x - hx^2 - l_y^2 . x + l_y . x^2 + l_y x^2 - x^3)dx$$

$$= \frac{q}{h . l_y} \left[\left\{ \frac{1}{2}(h . l_y - l_y^2)x^2 + \frac{1}{3}(2l_y - h)x^3 - \frac{1}{4}x^4 \right\} \right]_{(l_y-h)}^{l_y}$$

$$= \frac{q}{h . l_y} \left\{ \frac{1}{2}(h . l_y - l_y^2)\left[l_y^2 - (l_y - h)^2\right] \right.$$

$$\left. + \frac{1}{3}(2l_y - h)\left[l_y^3 - (l_y - h)^3\right] - \frac{1}{4}\left[l_y^4 - (l_y - h)^4\right] \right\}$$

$$= \frac{q}{h.l_y}\left\{ \frac{1}{2}(h\ l_y - l_y^2)\ (l_y^2 - l_y^2 + 2l_y h - h^2) + \frac{1}{3}(2\ l_y - h)\right.$$

$$\left[l_y^3 - l_y^3 + 3\ l_y^2 h - 3\ ly\ h^2 + h^3\right] - \frac{1}{4}\left[l_y^4 - l_y^4 + l_y^3 h\right.$$

$$\left.\left. - ly^2 h^2 + ly\ h^3 - h^4\right]\right\}$$

$$= \frac{q}{h.l_y}\left\{ \frac{1}{2}(h\ l_y - l_y^2)\ (2l_y h - h^2) + \frac{1}{3}(2\ l_y - h)\right.$$

$$\left.\left[3\ l_y^2 h - 3\ l_y\ h^2 + h^3\right] - \frac{1}{4}\left(l_y^3 h - l_y^2 h^2 + l_y\ h^3 - h^4\right]\right\}$$

$$= \frac{q}{h.l_y}\left\{ ly^2 h^2 - \frac{1}{2}\ l_y h^3 - l_y^3 - \frac{1}{2}\ ly^3 h^2 + 2\ l_y^3 h - 2l_y^2 h^2\right.$$

$$+ \frac{2}{3} l_y\ h^3 - l_y^2 h^2 + l_y\ h^3 - \frac{1}{3}\ h^4 - \frac{1}{4}\ l_y^2 h^2 - \frac{1}{4}\ l_y\ h^3$$

$$\left. + \frac{1}{4}\ h^4\right]\Big\}$$

$$= \frac{q}{h.l_y}\left\{ \frac{1}{12} h^4 + \left(-\frac{1}{2}\ l_y + \frac{2}{3}\ l_y + l_y - \frac{1}{4}\ ly\right) h^3\right.$$

$$+ (ly^2 + \frac{1}{2}\ l_y^3 - 2\ l_y^2 - l_y^2 + \frac{1}{4}\ l_y^2) h^2 + (-l_y^3 + 2\ l_y^3$$

$$\left. - \frac{1}{4}\ l_y^3) h\right\}$$

$$M = \frac{q}{l_y}\left\{ \frac{1}{12} h^3 + \left(\frac{17}{12}\ l_y\right) h^2 + \left(\frac{1}{2}\ l_y^3 - \frac{7}{4}\ l_y^2\right) h\right.$$

$$\left. + \left(\frac{3}{4}\ l_y^3\right)\right\}$$

$$l_y = c\ l_x$$

$$M = \frac{q}{\mu(1+i)}\left\{ -\frac{1}{12\ c\ l_x}\ h^3 + \frac{17}{12}\ h^2 + \left(\frac{1}{2}\ c^2 l_x^2 - \frac{7}{4}\ c\ l_x\right)\right.$$

$$\left. h + \frac{3}{4}\ c^2 l_x^2\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Persamaan (1) = Persamaan (2)

$$\frac{q}{8c^2(1+i)}\left[-\frac{1}{2}h^2+\left(\frac{1}{3}-c\,l_x\right)h+\left(\frac{1}{2}c^2\,l_x^2-\frac{2}{3}c\,l_x\right)\right]$$

$$= \frac{q}{\mu(1+i)}\left[-\frac{1}{12\,c\,l_x}h^3+\frac{17}{12}h^2-\frac{7}{4}c\,l_x)h+\frac{3}{4}c^2 l_x^2\right.$$

$$\frac{1}{16\,c^2}h^2+\frac{1}{8c^2}\left(\frac{1}{3}-c\,l_x\right)h+\frac{1}{8c^2}\left(\frac{1}{2}c^2\,l_x^2-\frac{2}{3}c\,l_x\right)$$

$$+\frac{1}{\mu 12\,c\,l_x}h^3-\frac{17}{12\mu}h^2-\frac{1}{\mu}\left(\frac{1}{2}c^2 l_x^2-\frac{7}{4}c\,l_x\right)h$$

$$-\frac{3}{4\mu}c^2\,l_x^2=0$$

$$\frac{1}{12\,\mu c l_x}h^3+\left(\frac{1}{16\,c^2}-\frac{17}{12\,\mu}\right)h^2+\left(\frac{1}{24\,c^2}-\frac{1}{8\,c}l_x\right.$$

$$-\frac{1}{2\,\mu}c^2\,l_x^2+\frac{7}{4\mu}c\,l_x)h+\frac{1}{8\,c}\left(\frac{1}{2}c\,l_x^2+\frac{3}{2}l_x\right.$$

$$-\frac{3}{4\,\mu}c^2 l_x^2=0$$

$$\frac{df}{dh}=0$$

$$\frac{1}{4\,\mu c l_x}h^2+\left(\frac{1}{8c^2}-\frac{17}{6\,\mu}\right)h+\left(\frac{1}{24\,c^2}-\frac{1}{8\,c}l_x\right.$$

$$\left.-\frac{1}{2\,\mu}c^2\,l_x^2+\frac{7}{4\mu}\,l_x\right)=0$$

akar positif

$$h=\frac{1}{8\mu\;c\;l_x}\left[\left(\frac{17}{64}-\frac{1}{8\;c^2}\right)\right.$$

$$\left.+\sqrt{\left(\frac{1}{8c^2}-\frac{17}{6\mu}\right)^2-\frac{1}{4\mu\;c\;l_x}\left[\frac{1}{24c^2}-\left(\frac{1}{8\;c}+\frac{7}{4\;\mu}-\frac{c\;l_x^2}{2\mu}\right)l_x\right]}\right]$$

Misal :

$$A = \sqrt{\left(\frac{1}{8\ c^2} - \frac{17}{6\mu}\right)^2 - \frac{1}{24\mu\ c^3 l_x}\left(\frac{1}{2\mu C} - \left(\frac{1}{4\ c} + \frac{7}{2\ \mu} - \frac{c^2 lx}{\mu}\right)\right)}$$

Maka :

$$h = \frac{1}{8\mu\ c l_x}\left(\frac{17}{6\mu} - \frac{1}{8\ c^2} + A\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Persamaan (3) disubtitusikan ke persamaan (1) diperoleh besarnya nilai M.